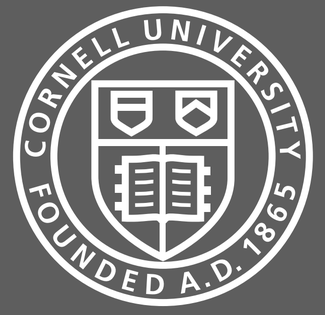
Distortion of Ballot Formats

Paul Gölz

Lecture 19, ORIE 4360/5360, Fall ’26



Distortion for Ballot Formats

Distortion of f on specific preference profile:

$$\mathrm{dist}(f, \boldsymbol{\sigma}) := \sup_{\boldsymbol{u} \rhd \boldsymbol{\sigma}} \frac{\max_{x \in M} \mathrm{SW}(x, \boldsymbol{u})}{\mathrm{SW}(f(\boldsymbol{\sigma}), \boldsymbol{u})}.$$

(back to nonnegative utilities, no metric.)

Distortion of f:

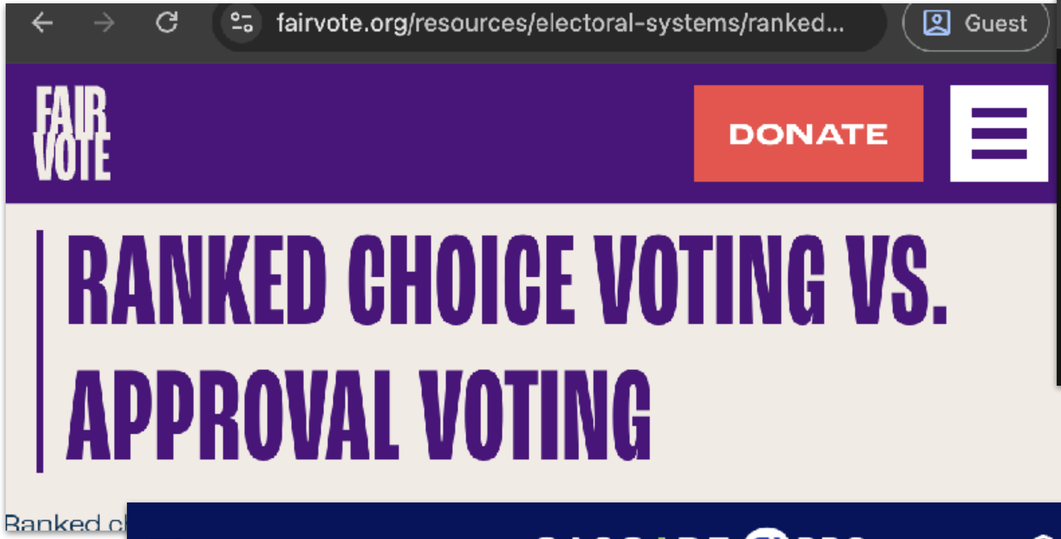
$$\mathrm{dist}(f) := \sup_{\boldsymbol{\sigma} \in \mathscr{L}^n} \mathrm{dist}(f, \boldsymbol{\sigma}).$$

"judge f by its worst performance on any $\boldsymbol{\sigma}$"

How can we define the unavoidable distortion when using ranked choice ballots?

$\mathrm{dist}_{\mathrm{ranking}} :=$

Comparing Ballot Formats









Comparing by Inherent Distortion

Know that (deterministic) $\text{dist}_{\text{ranking}} = \Theta(m^2)$. Measures how much welfare you might have to lose, no matter how good your voting rule.

Idea: determine inherent distortion of other ballot formats, lower is better.

Example: Single-Choice Ballots

Voter i just gives favorite candidate c_i on ballot.

"$\boldsymbol{u} \rhd \boldsymbol{c}$" means that $u_i(c_i) \geq u_i(x)$ for all i, x.

Then, distortion of rule $f : M^n \to M$:

$$\text{dist}(f, \boldsymbol{c}) := \sup_{\boldsymbol{u} \rhd \boldsymbol{c}} \frac{\max_{x \in M} \text{SW}(x, \boldsymbol{u})}{\text{SW}(f(\boldsymbol{c}), \boldsymbol{u})},$$

$$\text{dist}_{\text{single}} := \inf_{f: M^n \to M} \sup_{\boldsymbol{c}} \text{dist}(f, \boldsymbol{c}).$$

Still assume $u_i(x)$ are nonnegative, add to 1.

We know $\text{dist}_{\text{single}} = O(m^2)$ because plurality.

$\text{dist}_{\text{single}} = \Omega(m^2)$ since any single-choice rule can be implemented as a ranked-choice rule.

Example: Top-3 Approval

Voter i approves their top-3 alternatives. "$\boldsymbol{u} \rhd \boldsymbol{A}$" means that $u_i(x) \geq u_i(y)$ for all $i, x \in A_i, y \in M \backslash A_i$.

$$\text{dist}(f, \boldsymbol{A}) := \sup_{\boldsymbol{u} \rhd \boldsymbol{A}} \frac{\max_{x \in M} \text{SW}(x, \boldsymbol{u})}{\text{SW}(f(\boldsymbol{A}), \boldsymbol{u})},$$

$$\text{dist}_{\text{top-3}} := \inf_{f:\binom{M}{3}^n \to M} \sup_{\boldsymbol{A}} \text{dist}(f, \boldsymbol{A}).$$

Assume $u_i(x)$ are nonnegative, add to 1.

What is the distortion of top-3 approval?

pollev.com/goelz

You have selected **3 / 4** projects.

BLA Gym Renovations

No more netless hoops and broken bleachers--the BLA gym will be revamped to include newly painted hoops and floors, and repaired bleachers.

Estimated Cost: $475,000

Location: 205 Townsend Street, Roxbury

Select



Click image for slide show

Bowdoin St. Roadway Resurfacing

Bowdoin street from Geneva to Adams will be repaved. This will fix things like potholes, cracks, and faded roadway markings.

Estimated Cost: $100,000

Location: Bowdoin street from Geneva to Adams, Dorchester

✔ Selected | Remove



Click image for slide show

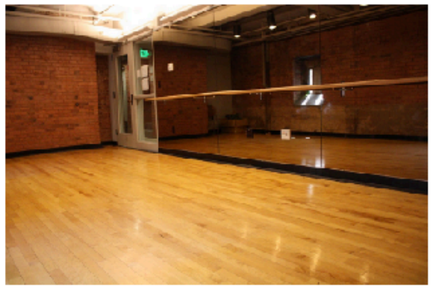
BCYF Hyde Park Dance Studio Renovation

A renovated dance studio at the Hyde Park Community Center for childr all ages.

Estimated Cost: $286,000

Location: BCYF Hyde Park Community Center, Hyde Park

Select



Click image for slide show

Ringer Park Renovation

Renovate the main path in Ringer Playground so it is no longer cracked and safer to walk on.

Estimated Cost: $280,000

Location: Ringer Park, Brighton

✔ Selected | Remove



Wicked Free Wifi 2.0

Wicked free Wi-Fi 2.0 provides Wi-fi at locations with young people.

Estimated Cost: $119,000

Location: Various High Schools and Community Centers, Dorchester, Roxbury, East Boston, Charlestown

✔ Selected | Remove



Track at Walker Park

Build a track that goes around the field that is behind Walker Playground.

Estimated Cost: $240,000

Location: Walker Playground on Norfolk St. and Astoria St., Mattapan

Select



"Youth Lead the Change" PB in Boston, $1M budget in 2015.
Source: https://pbstanford.org/boston15/approval

Distortion for Participatory Budgeting

Let’s revisit participatory budgeting!

So far, we assumed approval ballots, but voters ranking projects also very common.



Participatory Budgeting Model

Voters $N = \{1,2,\ldots,n\}$. Alternatives/projects M, $|M| = m$.

Budget $B > 0$, and cost $0 < c_x \leq B$ for each alternative x.

Let $\mathcal{F}$ denote the set of feasible outcomes

$\mathcal{F} = \{W \subseteq M \mid \sum_{x \in W} c_x \leq B\}$.

Each voter i has a value $v_i(x) \geq 0$ for each project x. Assume normalization $\sum_{x \in M} v_i(x) = 1$.

Voter i has additive utility $u_i(W) := \sum_{x \in W} v_i(x)$ for sets W of projects. Utilitarian welfare $\mathrm{SW}(W, \boldsymbol{v}) := \sum_{i \in N} u_i(W)$.

How Do People Vote in PB?

Approving projects:

☑🚲 ☐🌳 ☑🛝

Ranking projects:

🚲 ≻ 🛝 ≻ 🌳

How do ballots reflect utilities?

- **Knapsack:** Voter approves feasible set with maximum utility.
- **Top-t:** Voter approves t projects with largest values.
- **t-Threshold:** Voter approves all projects with value $\geq t$.

more informative

- **Rank-by-value:** Voter ranks projects in order of $v_i(x)$.
- **Bang-per-buck:** Voter ranks projects in order of $v_i(x)/c_x$.

Which one do you favor?

Knapsack Voting Example

Voter i votes in PB election with $B = \$24$:

project	a	b	c	d	e
cost	\$12	\$12	\$8	\$6	\$3
v_i	5	6	3	2	3

For knapsack ballot format, which set of projects does i approve?

pollev.com/goelz

Distortion of Voting Rules

For each of these ballot formats, can compute the distortion of a social choice function f:

$$\text{dist}(f) = \sup_{A, v \rhd A} \frac{\max_{W \in \mathcal{F}} \text{SW}(W, v)}{\text{SW}(f(A), v)} \text{ (for approval),}$$

$$\text{dist}(f) = \sup_{\sigma, v \rhd \sigma} \frac{\max_{W \in \mathcal{F}} \text{SW}(W, v)}{\text{SW}(f(\sigma), v)} \text{ (for rankings).}$$

Consistency depends on whether knapsack, top-t_i, …

If voting rule is randomized, expected social welfare.

Distortion of Ballot Formats

The distortion of a ballot format (e.g., knapsack approvals, rank-by-value, …) is the smallest distortion achieved by any social choice function on this ballot format, e.g., for ranking formats:

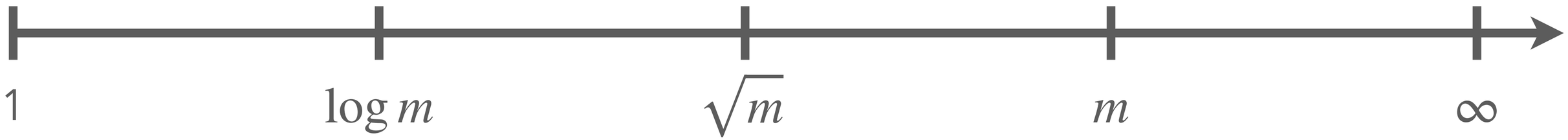
$$\text{dist}_{\text{format}} := \inf_{f:\mathscr{L}^n \to \Delta(M)} \text{dist}(f).$$



Allow randomization this time!

Distortion Landscape

Goal: Place the ballot formats on this line!



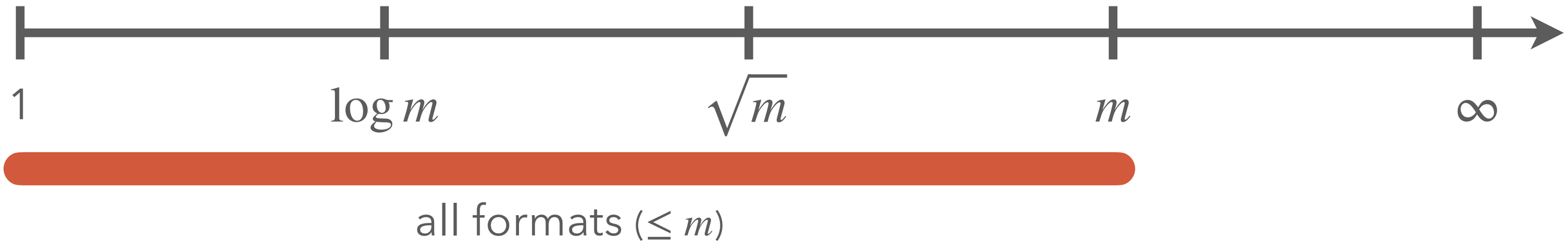
Upper Bound for all Voting Formats

Let $f_{\text{rand-alt}}$ be the social choice function that selects a single alternative, uniformly at random.

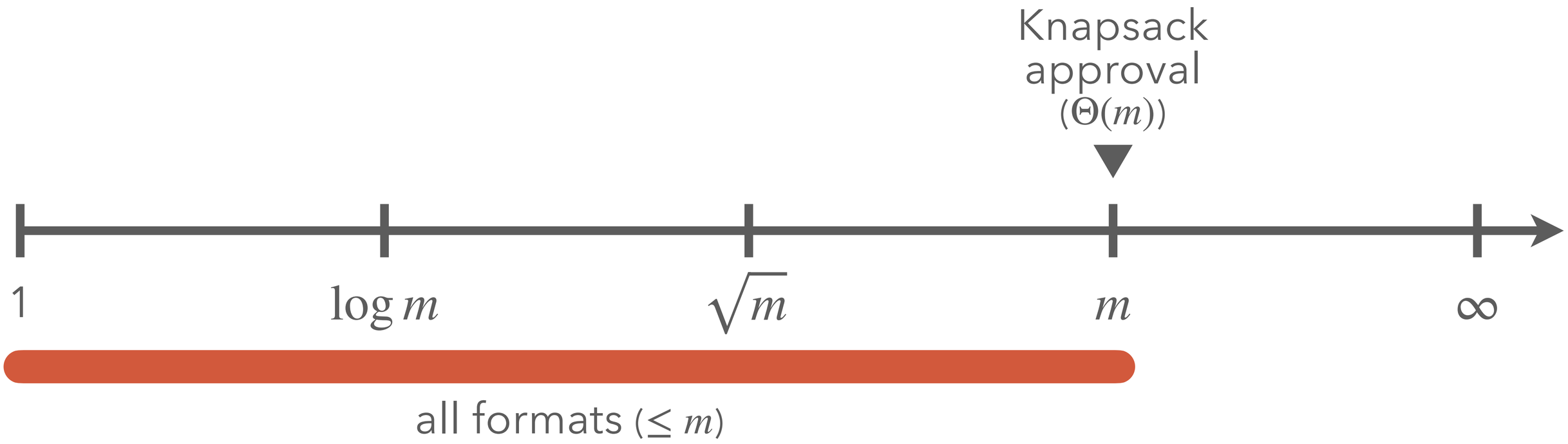
Observation: Since $f_{\text{rand-alt}}$ ignores its input, it is a social choice function for any ballot format!

Theorem (Benadè et al. 2021)**.** Any ballot format has distortion at most m.

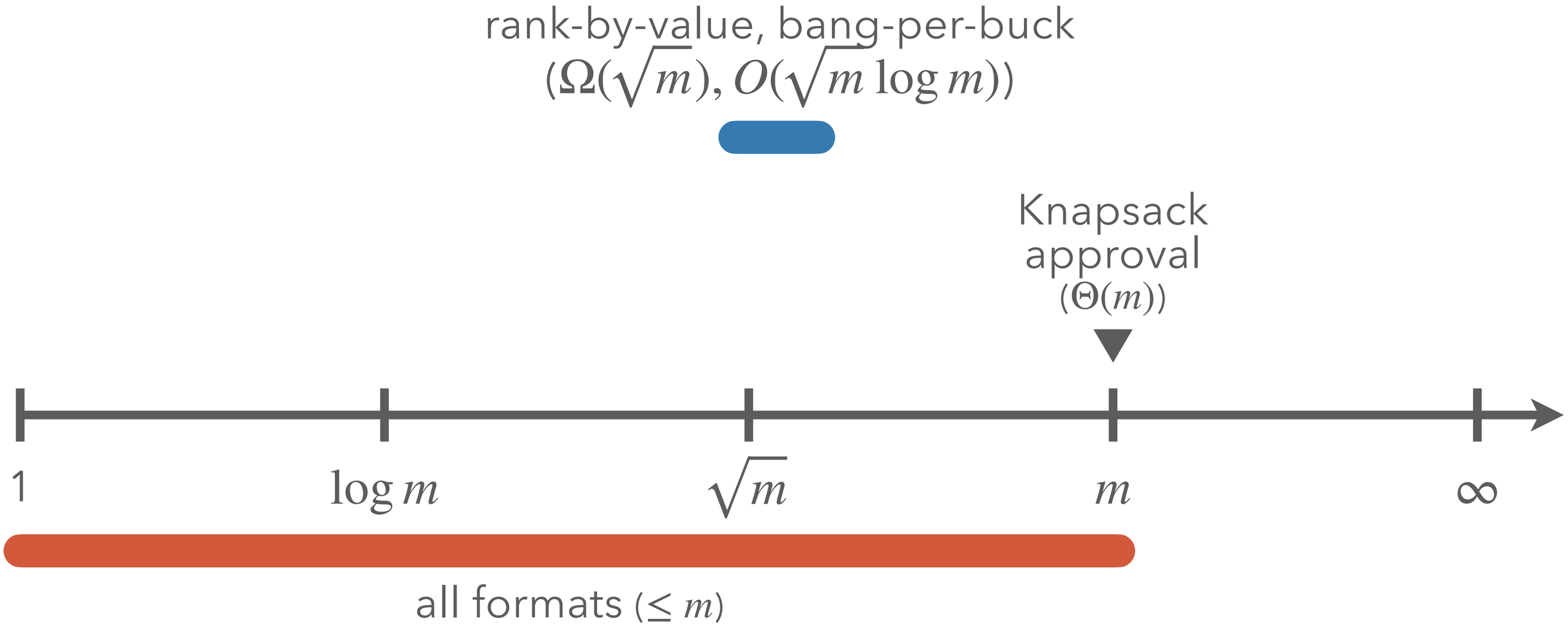
Distortion Landscape



Distortion Landscape



Distortion Landscape

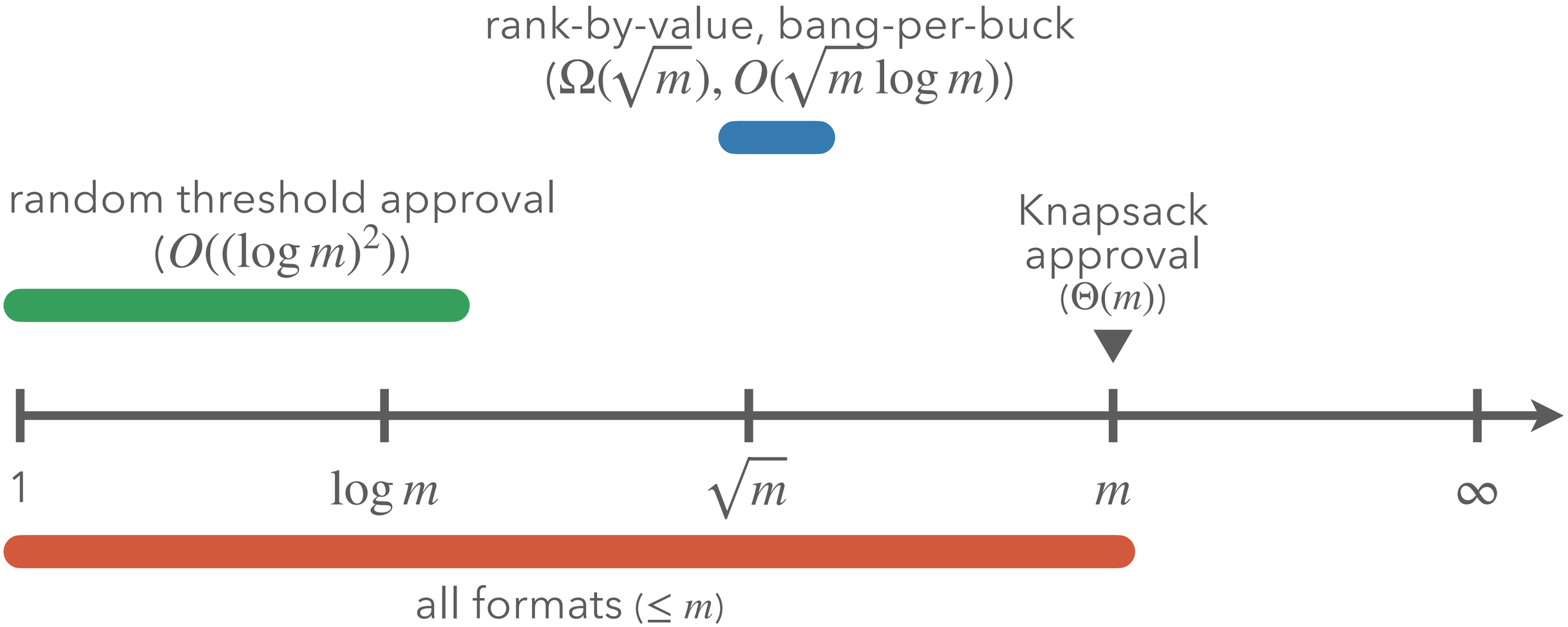


Threshold Approval Format

If single threshold is deterministically chosen, distortion is still at least $\Omega(\sqrt{m})$.

Theorem (Benadè et al. 2021)**.** If the thresholds are randomly set in the right way, the distortion of threshold approval is at most $O((\log m)^2)$.

Distortion Landscape



Try Out Some Ballot Formats!

You are stranded on a remote island. There are 10 items which may increase your chances of survival. Each item has a cost, and there is a budget of $65.

- Mirror ($10)
- Map ($8)
- Pocket knife ($5)
- Pistol ($30)
- Compass ($5)
- Sunglasses ($25)
- First aid kit ($10)
- Water ($3)
- Top coat ($20)
- Raincoat ($10)

Vote with rank-by-value, knapsack, and bang-per-buck!

tinyurl.com/4360-island

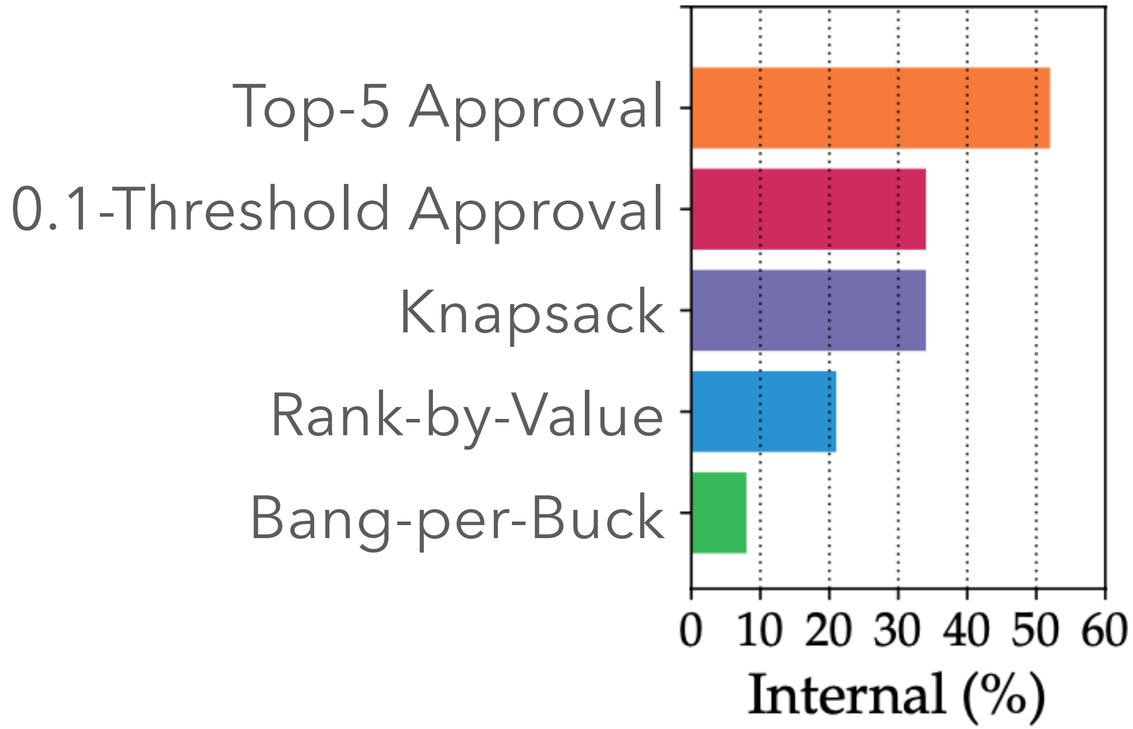
Discuss With Your Partner

Among the three ways of reporting your preferences:

- which one was easiest?
- which left you most confident that you gave the right answer (based on your own preferences)?
- which captured your preferences best?

Consistency for Online Participants

Benadè et al (2018) asked participants to directly report their values. How often are responses to format consistent with values?



Response Times for Online Participants

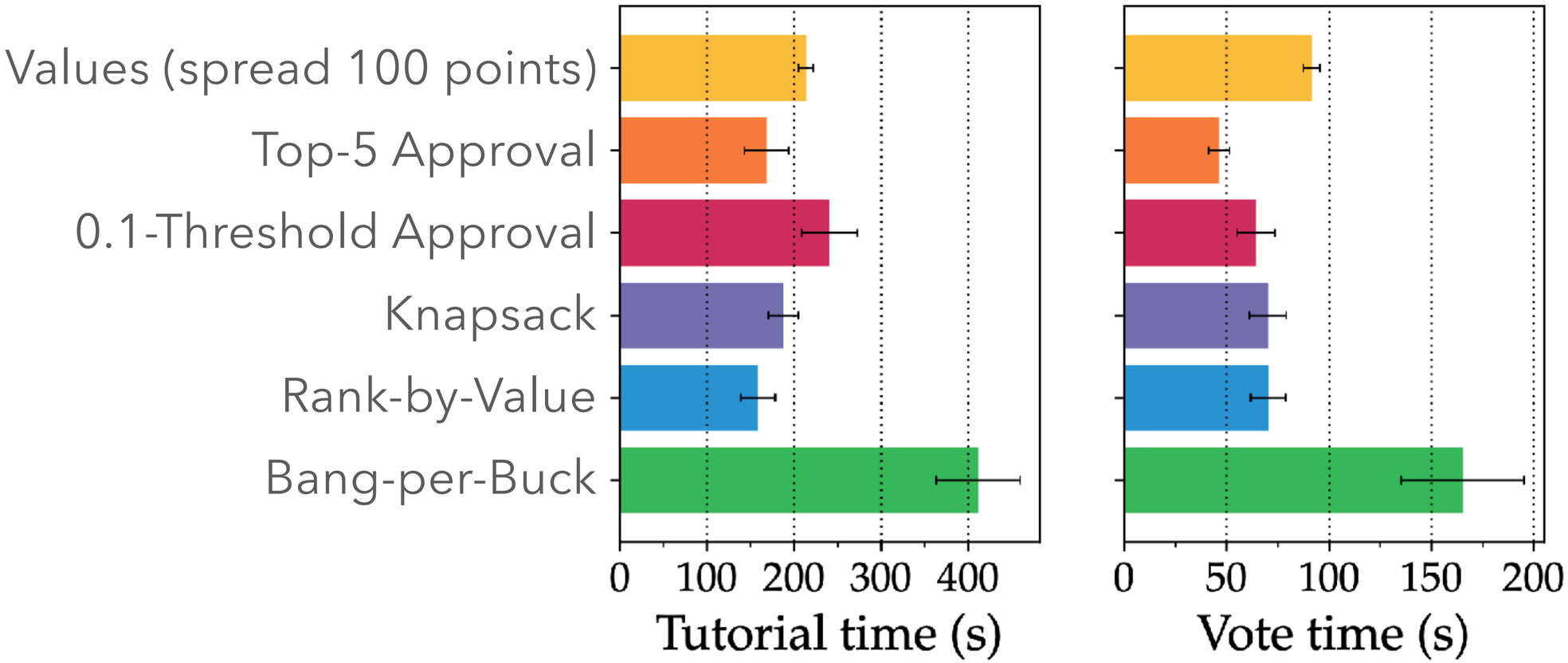
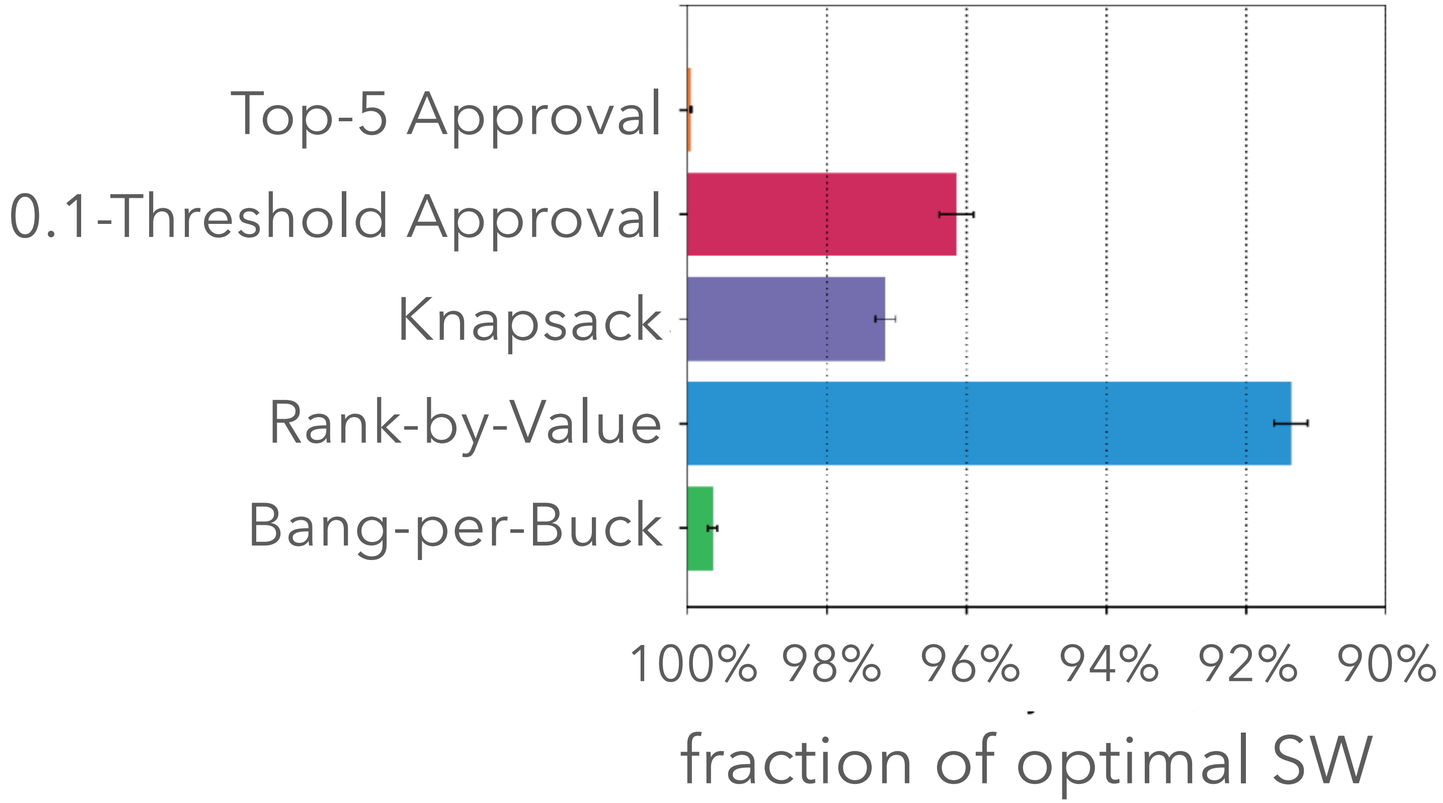


Figure 4: Average time taken (in seconds) to complete the pre-task tutorial and to cast a vote in each input format. Lower is better.

Distortion in Practice?

Run the social choice function that minimizes distortion on each voting format:



All ratios much better than distortion bounds.

Bang-per-buck & Top-t Approval great, Rank-by-Value bad.

Limitations:
- Unnatural voting rule
- Maybe limited to island scenario

Bibliography

Benadè et al.: *Preference elicitation for participatory budgeting*. Management Science (2021).

Benadè et al.: *Efficiency and Usability of Participatory Budgeting Methods*. Manuscript (2018).

Bedaywi et al.: *The Distortion of Public Spirited Participatory Budgeting*. AAAI 2025.