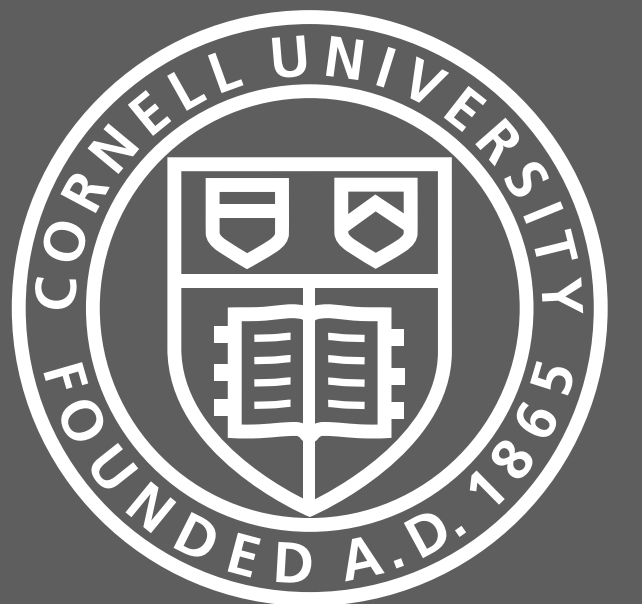


Participatory Budgeting

Paul Gözl

Lecture 14, ORIE 4360/5360, Fall '26



Recap: Extended Justified Repr. (EJR)

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Definition. For $1 \leq \ell \leq k$, a group $S \subseteq N$ is called ℓ -cohesive if $|\bigcap_{i \in S} A_i| \geq \ell$.

Definition.* A committee W satisfies **extended justified representation (EJR)** if there are no ℓ -cohesive groups S of size $\geq \ell \cdot n/k$ such that $u_i(W) < \ell$ for all $i \in S$.

A social choice function satisfies **EJR** if it returns, for any input profile, an EJR committee.

* Slightly rephrased, but equivalent to last lecture.

You have selected **3 / 4** projects.

BLA Gym Renovations

No more netless hoops and broken bleachers--the BLA gym will be revamped to include newly painted hoops and floors, and repaired bleachers.

Estimated Cost: \$475,000

Location: 205 Townsend Street, Roxbury

Select



Click image for slide show

Bowdoin St. Roadway Resurfacing

Bowdoin street from Geneva to Adams will be repaved. This will fix things like potholes, cracks, and faded roadway markings.

Estimated Cost: \$100,000

Location: Bowdoin street from Geneva to Adams, Dorchester

✓ Selected

Remove



Click image for slide show

BCYF Hyde Park Dance Studio Renovation

A renovated dance studio at the Hyde Park Community Center for children of all ages.

Estimated Cost: \$286,000

Location: BCYF Hyde Park Community Center, Hyde Park

Select



Click image for slide show

Ringer Park Renovation

Renovate the main path in Ringer Playground so it is no longer cracked and safer to walk on.

Estimated Cost: \$280,000

Location: Ringer Park, Brighton

✓ Selected

Remove



Wicked Free Wifi 2.0

Wicked free Wi-Fi 2.0 provides Wi-fi at locations with young people.

Estimated Cost: \$119,000

Location: Various High Schools and Community Centers, Dorchester, Roxbury, East Boston, Charlestown

✓ Selected

Remove



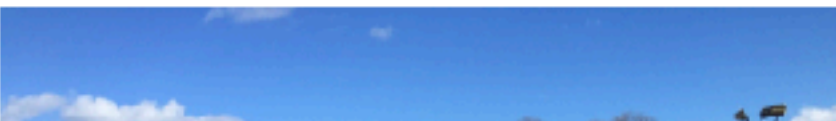
Track at Walker Park

Build a track that goes around the field that is behind Walker Playground.

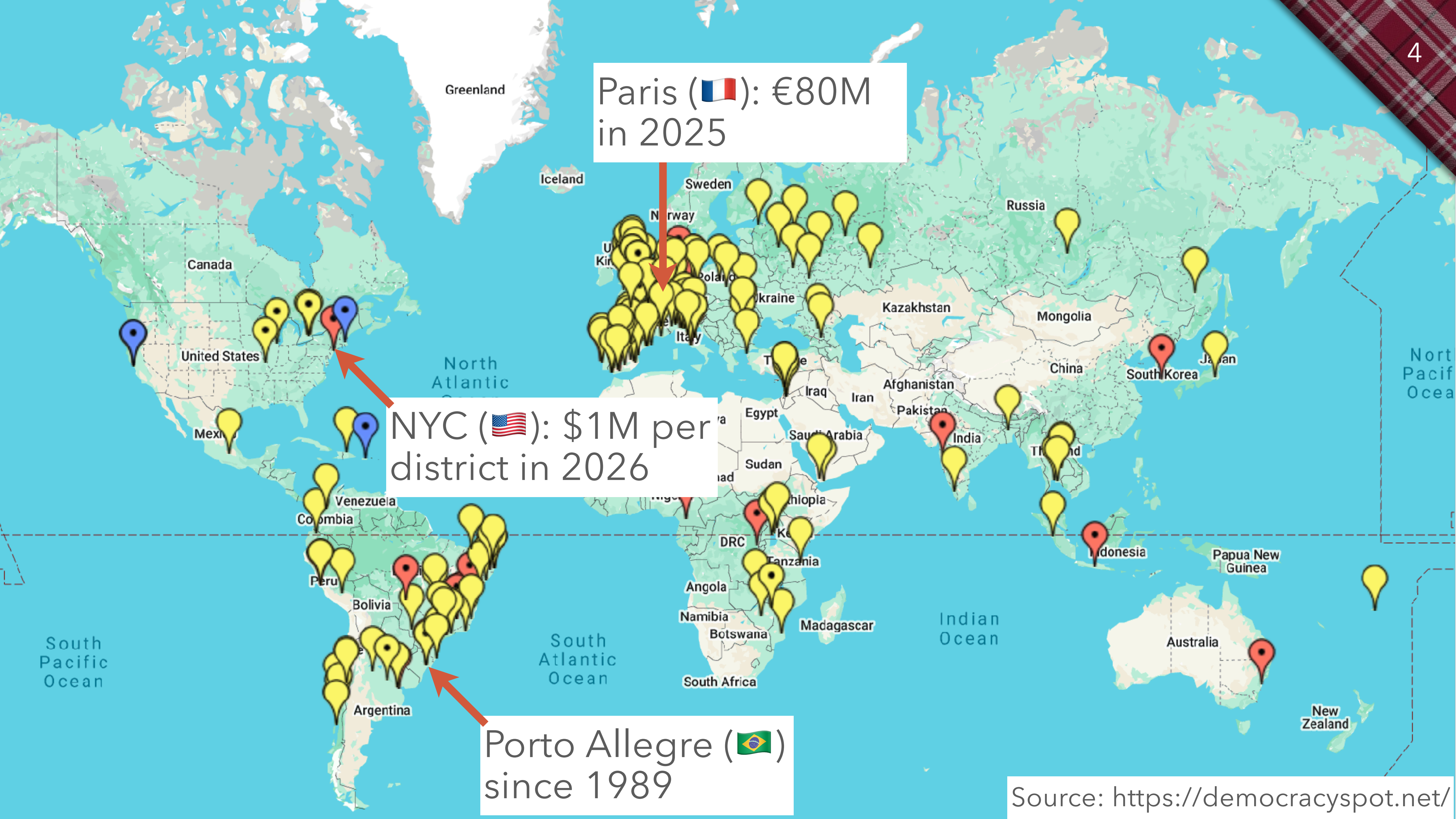
Estimated Cost: \$240,000

Location: Walker Playground on Norfolk St. and Astoria St., Mattapan

Select



“Youth Lead the Change” PB in Boston, \$1M budget in 2015.
Source: <https://pbstanford.org/boston15/approval>



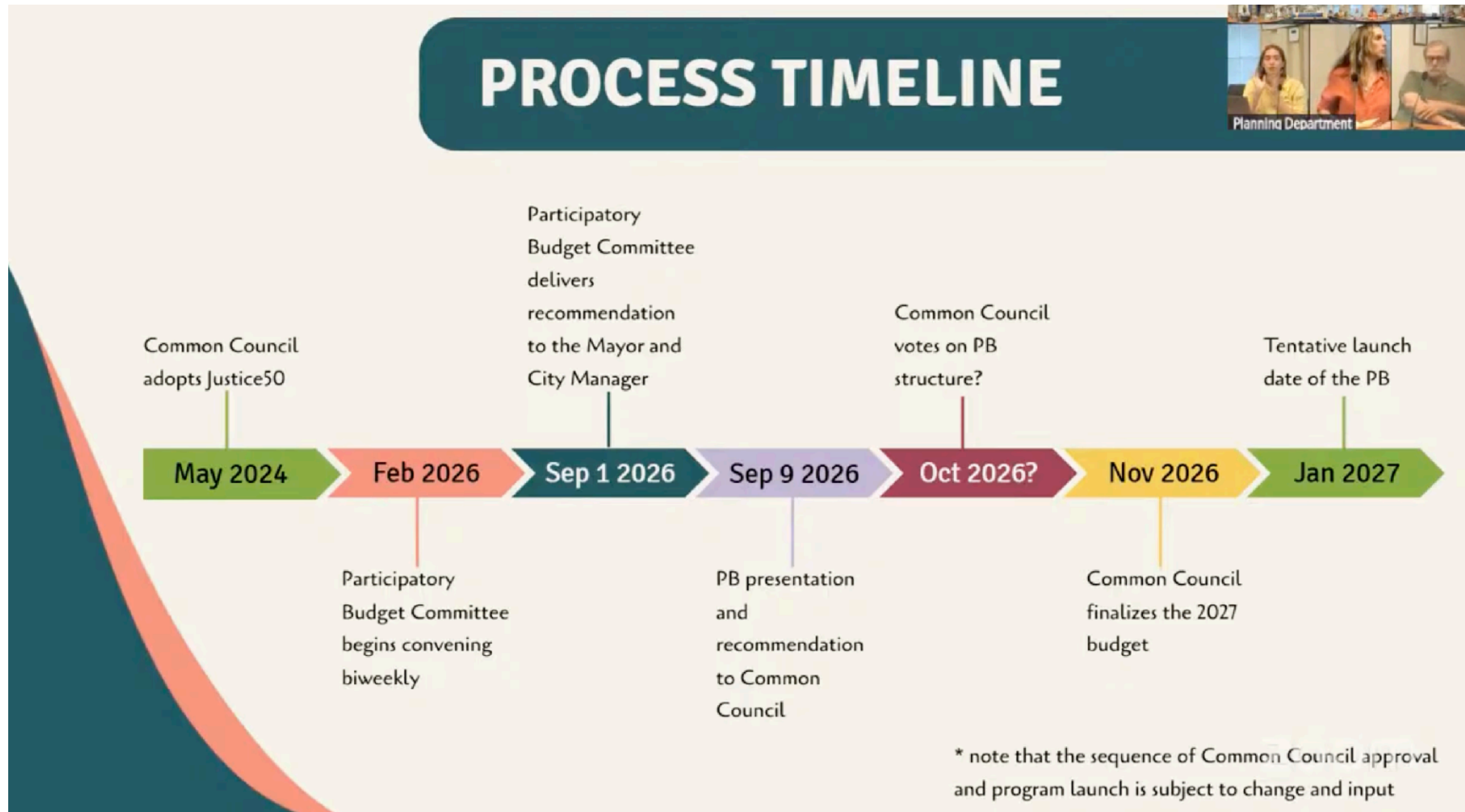
Paris (🇫🇷): €80M
in 2025

NYC (🇺🇸): \$1M per
district in 2026

Porto Alegre (🇧🇷)
since 1989

Coming to Ithaca?

5



Source: <https://www.youtube.com/watch?v=FQGYPr9CWpg>

Participatory Budgeting Voting Model

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Voters $N = \{1, 2, \dots, n\}$. Alternatives/projects M , $|M| = m$.

Each voter i has an approval set $A_i \in \mathcal{P}^*(M)$ of projects; preference profile $A \in \mathcal{P}^*(M)^n$ is a collection of all voters' approval sets.

Budget $B > 0$, and cost $c_x > 0$ for each alternative x . Let $\mathcal{F}$ denote the set of feasible outcomes

$$\mathcal{F} = \{W \subseteq M \mid \sum_{x \in W} c_x \leq B\}.$$

Social choice function is a function $f: \mathcal{P}^*(M)^n \rightarrow \mathcal{F}$, decides which projects to fund.

PB and Multi-Winner Elections

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Budget $B > 0$, and cost $c_x > 0$ for each alternative x . Let $\mathcal{F}$ denote the set of **feasible outcomes**

$$\mathcal{F} = \{W \subseteq M \mid \sum_{x \in W} c_x \leq B\}.$$

Observation: If we set all costs to 1 and $B = k$,

$\mathcal{F} = \binom{M}{k} \cup \binom{M}{k-1} \cup \dots \cup \binom{M}{0}$. Shows that participatory budgeting setting generalizes multi-winner elections (except that technically allowed to select $< k$ alternatives).

Can you think of any social choice function for the participatory budgeting?

Generalizing Multi-Winner Rules

Approval voting (AV):

$$f_{AV}(A) := \operatorname{argmax}_{W \in \mathcal{F}} \sum_{i \in N} |A_i \cap W|.$$

Proportional Approval Voting (PAV):

$$f_{PAV}(A) := \operatorname{argmax}_{W \in \mathcal{F}} \sum_{i \in N} H_{|A_i \cap W|}.$$

$$H_t = \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{t}$$

In practice, people often do *greedy AV*, even simpler: Go through projects from highest approval score to lowest. If still fits in the budget, fund it; else, skip to next.

Paris 2019 Example (4e Arr.)

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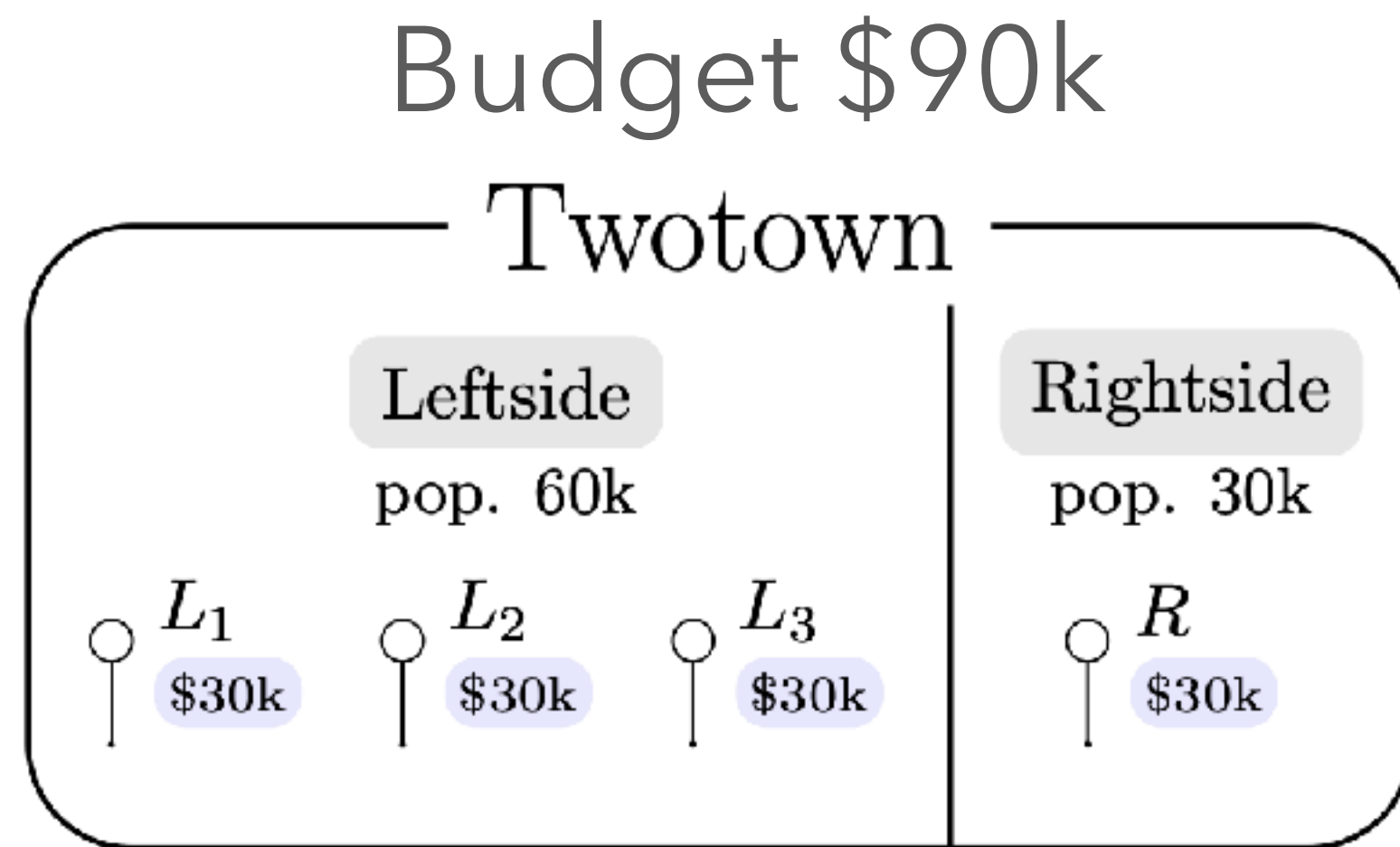
Budget: €1,465,000

Greedy
AV AV

Project Name	QPOP	Cost €	Votes	v / k€	Greedy	Optimal
 Un mur végétalisé au croisement des rues Blancs Manteaux et Archiv		30 000	788	26	✓	✓
 Un café solidaire dans le quartier de la tour Saint-Jacques		15 000	706	47	✓	✓
 Une salle d'arts plastiques pour l'école Saint-Merri Renard		300 000	702	2	✓	✓
 Rénovation énergétique exemplaire d'une école du 4e		1 000 000	655	1	✓	—
 Végétalisation de la rue de l'Arsenal		120 000	649	5	✓	✓
 Un collège Charlemagne accessible aux personnes à mobilité réduit		200 000	630	3	—	✓
 Faire du hall d'accueil de la piscine Saint-Merri un lieu de convivia		20 000	528	26	—	✓
 Des agrès sportifs place des Vosges		15 000	491	33	—	✓
 Mise en valeur des pierres de la prison de la Bastille		20 000	473	24	—	✓
 Un fauteuil roulant électrique pour rompre l'isolement		5 000	453	91	—	✓
 Création d'un auvent sur une aire de jeux d'un square du 4e		150 000	410	3	—	✓
 L'Ascenseur, un tiers-lieu pour l'égalité des chances ouvert sur le		350 000	315	1	—	✓
 Baliser le passage du chemin de Compostelle dans le 4e		30 000	265	9	—	✓
 Des arceaux vélos rue de la Reynie		10 000	240	24	—	✓
 Faciliter l'accès au cellier d'Ourscamp		120 000	228	2	—	✓
Total €					1 465 000	1 385 000

Is PAV Still Proportional?

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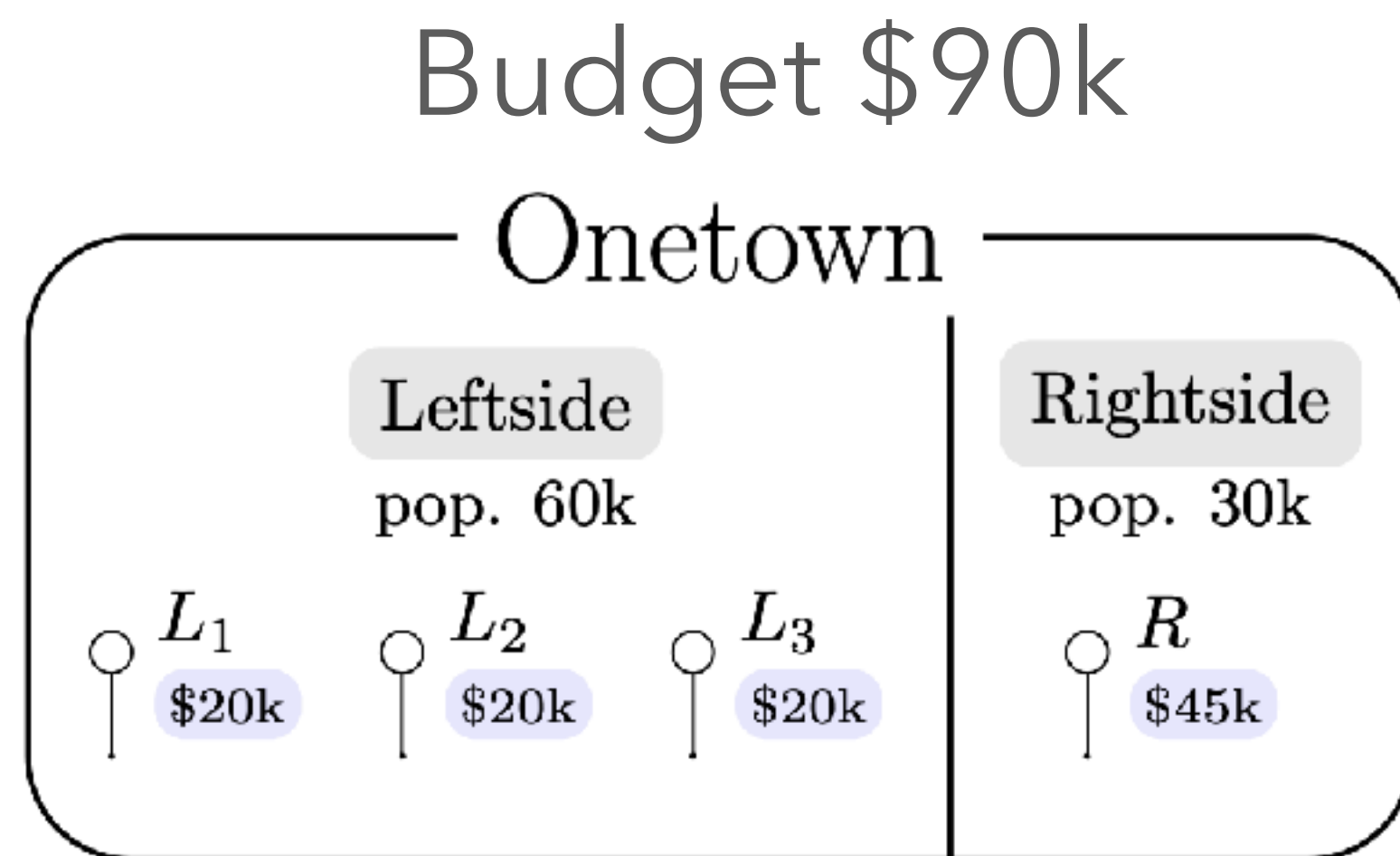
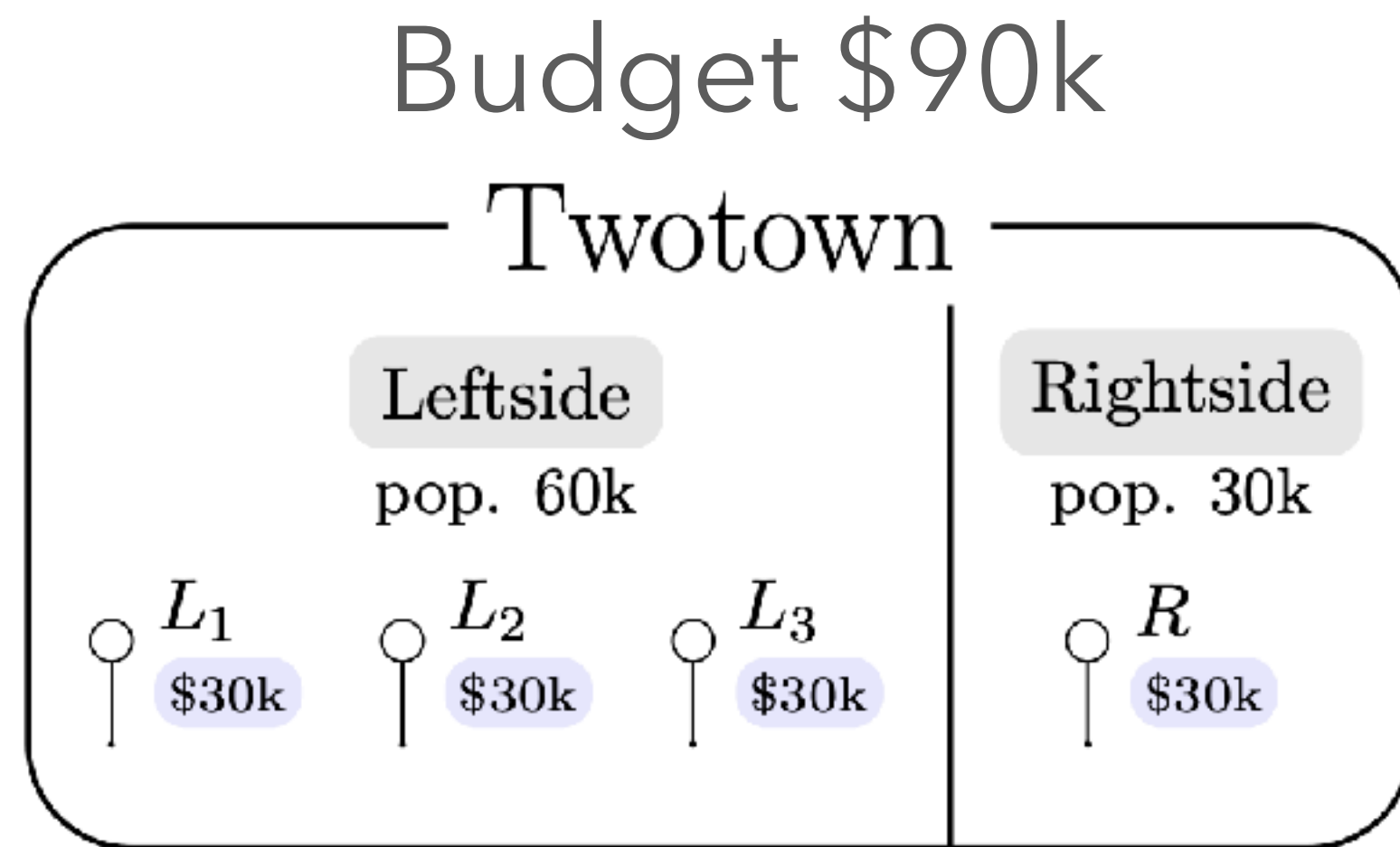
$$f_{\text{PAV}}(A) := \operatorname{argmax}_{W \in \mathcal{F}} \sum_{i \in N} H_{|A_i \cap W|}$$

$$H_t = \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{t}$$

Which projects does PAV choose in Twotown?

PAV Fails EJR

11



$$f_{\text{PAV}}(A) := \operatorname{argmax}_{W \in \mathcal{F}} \sum_{i \in N} H_{|A_i \cap W|}$$

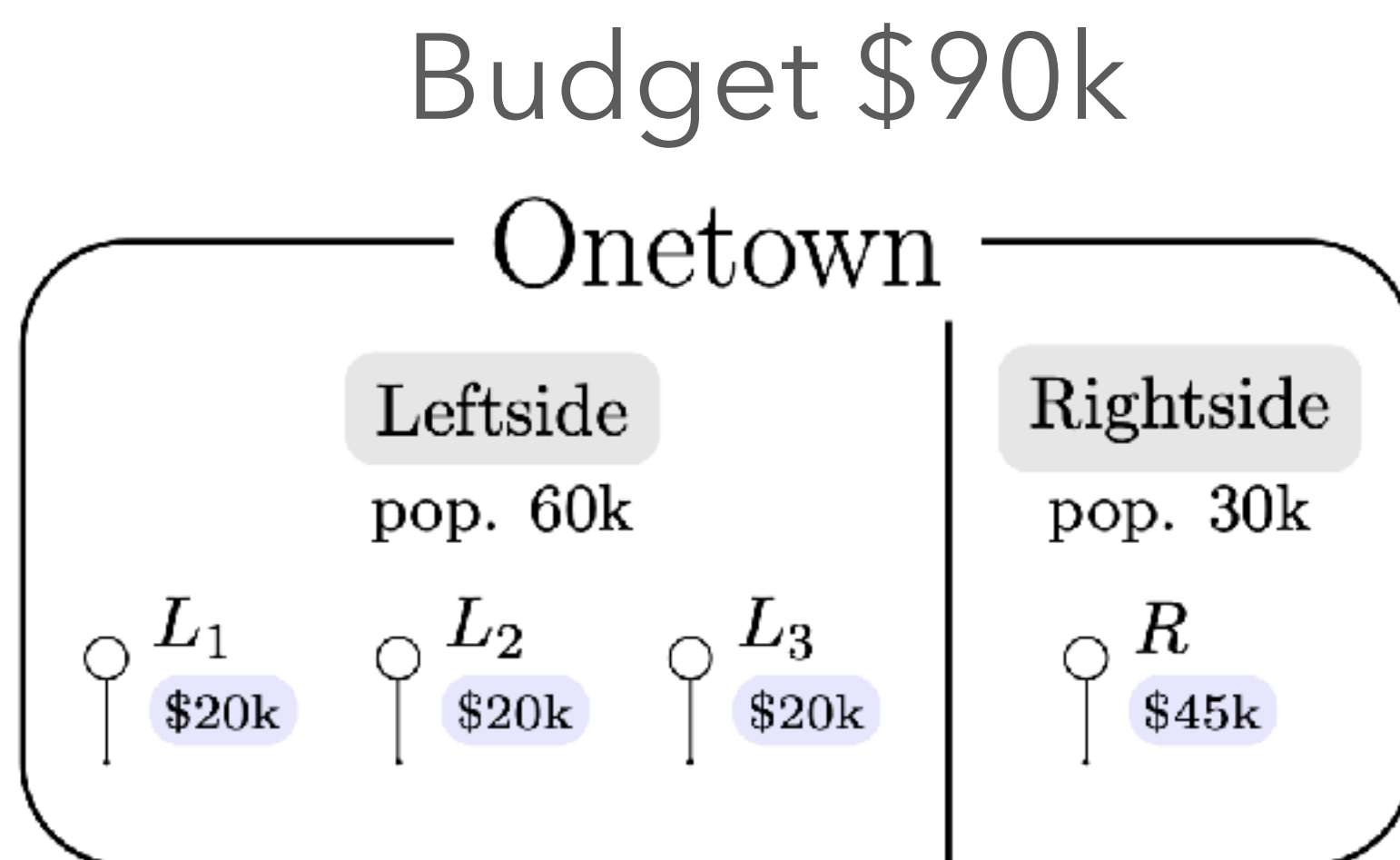
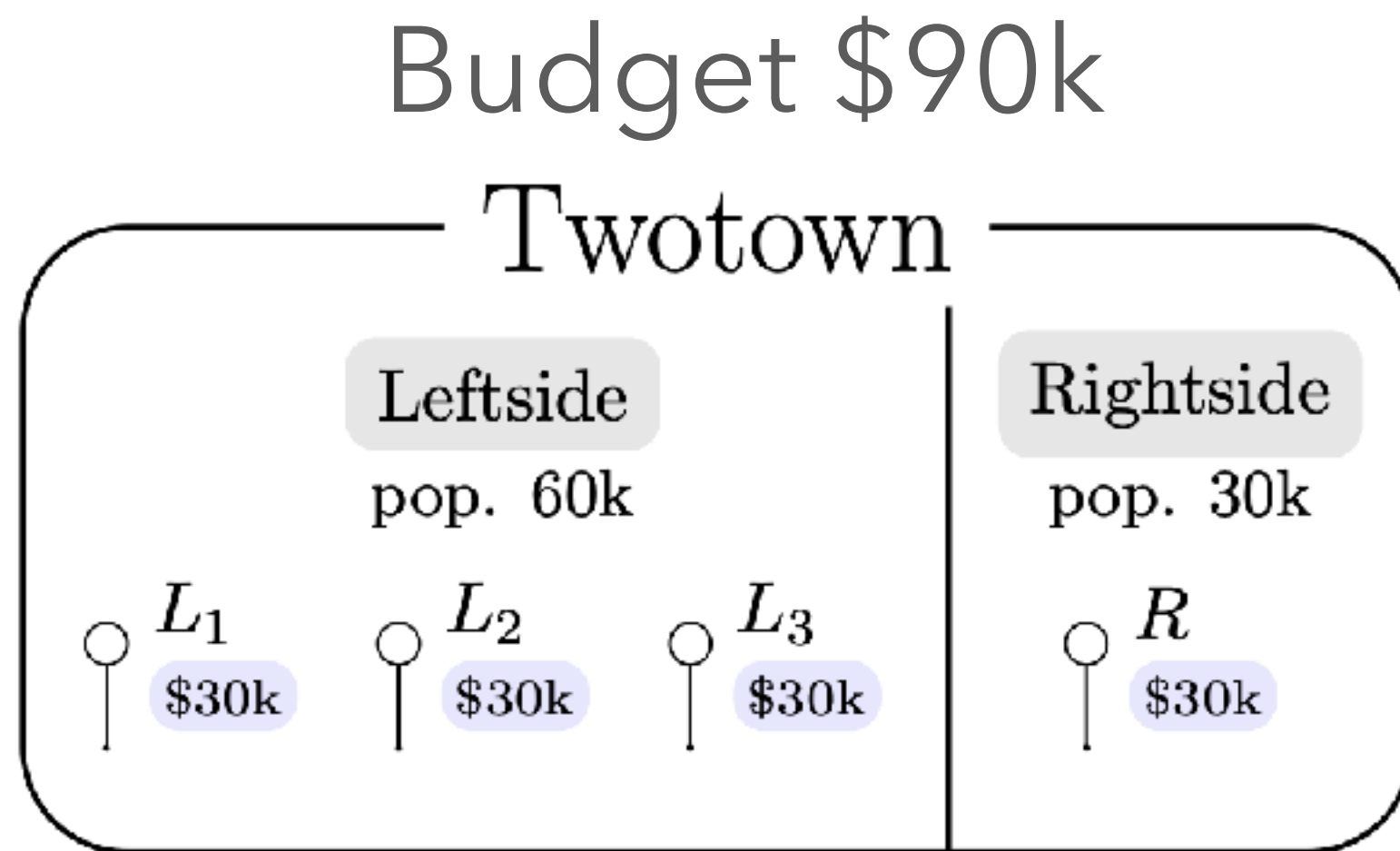
$$H_t = \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{t}$$



Which projects
does PAV choose in
Onetown?

pollev.com/goelz

PAV Fails EJR



$$f_{\text{PAV}}(A) := \operatorname{argmax}_{W \in \mathcal{F}} \sum_{i \in N} H_{u_i}(W)$$

$$H_t = \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{t}$$

PAV does not see the costs, but the costs change what is proportional.

Let's Generalize EJR

Definition. An outcome W satisfies **extended justified representation (EJR)** if there is no group $S \neq \emptyset$ and set T of projects such that

- $T \subseteq \bigcap_{i \in S} A_i$
- $|S| \geq n \cdot (\sum_{x \in T} c_x) / B$
- "all $i \in S$ are **happier** with W than T ".

How To Define “Happier”?

In multi-winner elections, only natural choice was counting approved projects: $u_i(W) = |A_i \cap W|$.

In PB, this is one option, **cardinality utilities**.

Other natural option: more expensive approved projects make voter happier. **Cost utilities**:

$$u_i(W) := \sum_{x \in A_i \cap W} c_x.$$

Will define EJR for either option.

Definition. An outcome W satisfies **extended justified representation (EJR)** if there is no group $S \neq \emptyset$ and set T of projects such that

- $T \subseteq \bigcap_{i \in S} A_i$
- $|S| \geq n \cdot (\sum_{x \in T} c_x) / B$
- for all $i \in S$, $u_i(W) < u_i(T)$.

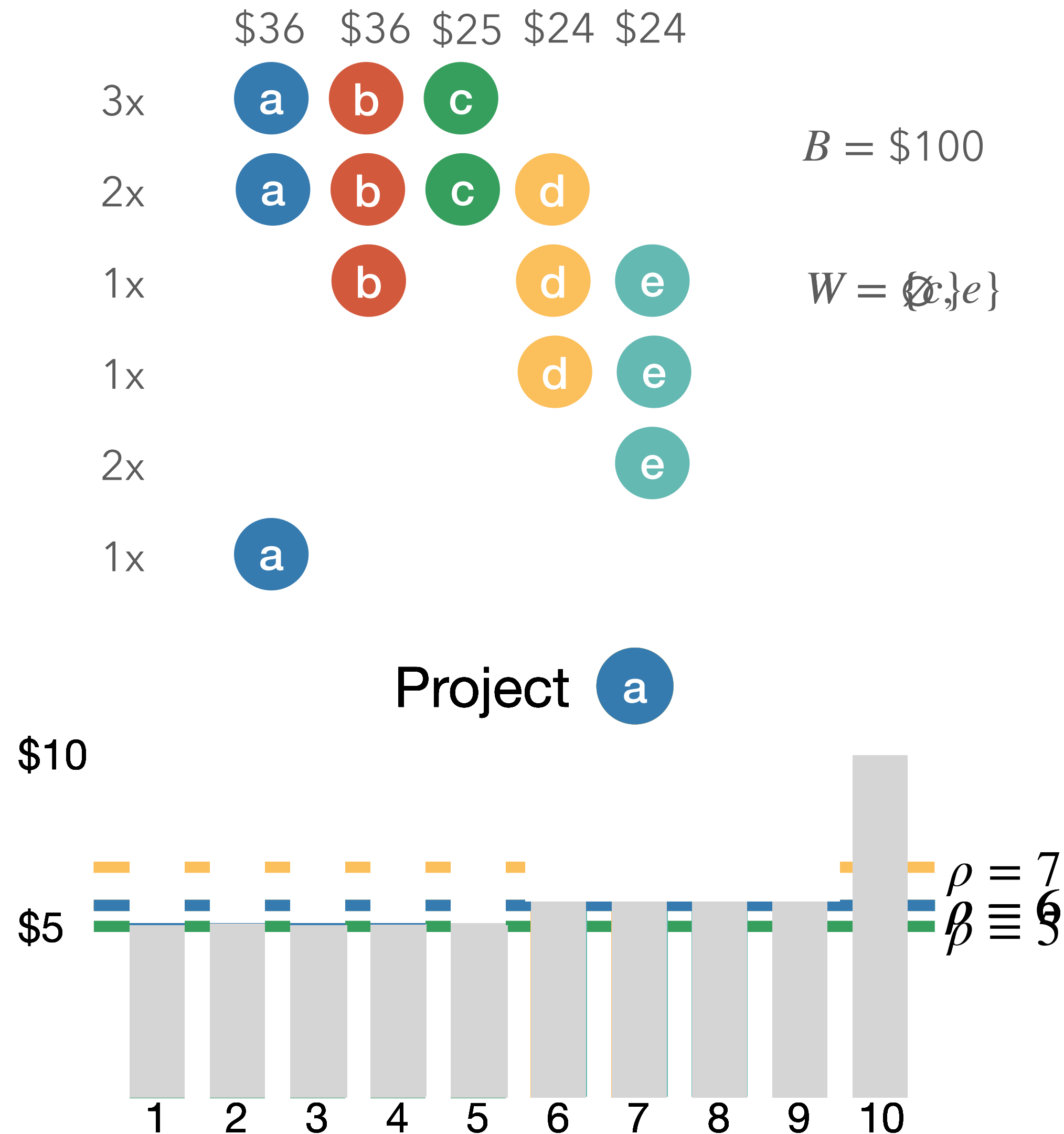
Works for cardinality utilities. For cost utilities, must weaken:

$$u_i(W) \leq u_i(T) - \max_{x \in T} c_x.$$

"EJR up to one project".

Method of Equal Shares

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Each voter is given initial budget $\beta_i = B/n$.

For $\rho \in \mathbb{R}$, alternative x is ρ -affordable if $\sum_{i \in N} \min\{\beta_i, \rho \cdot u_i(\{x\})\} \geq c_x$.

Let $W = \emptyset$. While there exists ρ -affordable alternative for some ρ :

- Let $x \in A$ be ρ -affordable for minimum ρ .
- Set $\beta_i := \beta_i - \min\{\beta_i, \rho \cdot u_i(\{x\})\}$ for all i .
- Add x to W .

Theorem (Peters et al. 2021). MES satisfies EJR for cardinality utilities and EJR up to one project for cost utilities.

Compute Method of Equal Shares!

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Project	Costs												vote count
 bike path	\$700	×	×	×	×	×	×					×	7
 outdoor gym	\$400	×	×	×	×	×	×						6
 new park	\$250		×		×	×		×			×		5
 new playground	\$200							×	×	×	×		4
 library for kids	\$100							×		×	×		3

$B = \$1100$, cost utilities

Method of Equal Shares

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Only proportional rule used in practice:



Major issue: Often doesn't spend entire budget.
Suggested approaches:

- Run for higher budget.
- Allocate remaining budget using other rule.

Bibliography

Peters et al.: *Proportional Participatory Budgeting with Additive Utilities*. NeurIPS (2021).