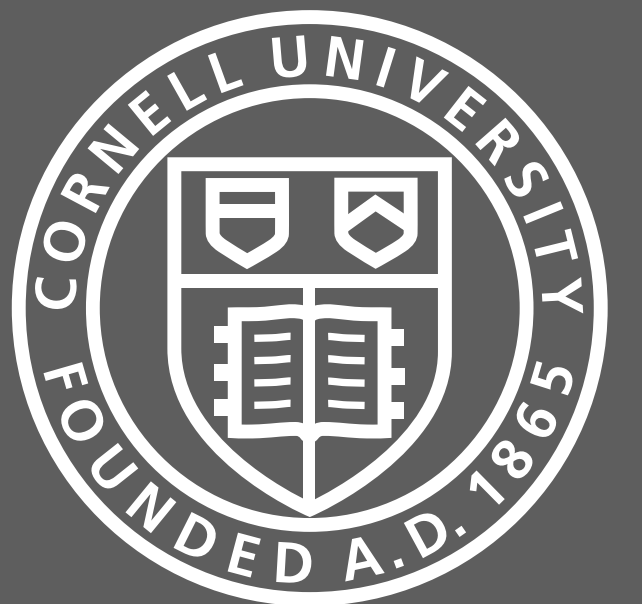


Liquid Democracy

Paul Gözl

Lecture 7, ORIE 4360/5360, Fall '26



Recap: Condorcet's Jury Theorem

2

2 alternatives: ⁺¹~~*a*~~ (good) and ⁻¹~~*b*~~ (bad).

Each of n voters independently votes for *a* with probability $1/2 < p \leq 1$ or for *b* with probability $1 - p$.



Nicolas de
Condorcet
(1743–1794)

Theorem [Condorcet 1785]. As $n \rightarrow \infty$, the probability that the majority is correct goes to 1.

Different Competences

Today, we will stay in this $m = 2$ setting.

In Condorcet's model, each voter has the same competence p , i.e., probability of identifying $+1$.

What if each voter i has a personalized competence $0 < p_i < 1$?

Example



Majority vote is correct with $\approx 77\%$ probability.

Competent person is correct with 80%.

Majority vote where competent person counts triple is correct with $\approx 81\%$.

Weighted Majority Rule

A social choice function maps a preference profile $\mathbf{v}$ to winner, $f: \{-1, +1\}^n \rightarrow \{-1, +1\}$.

Majority rule: choose $+1$ if more voters vote for $+1$ than -1 , i.e., if $\sum_{i \in N} v_i > 0$.

Weighted majority for weights $\mathbf{w} = (w_1, \dots, w_n)$: choose $+1$ if $\sum_{i \in N} w_i \cdot v_i > 0$.

Poll

6



$\frac{2}{3} \approx 67\%$



$\frac{16}{17} \approx 94\%$

+ other people...



To maximize the chance of being right, how much more should we weight the person on the right?

pollev.com/goelz

The Best Voting Rule?

7

Let $V \in \{-1, +1\}^n$ be random pref. profile.

“Best” = want to maximize chance that right alternative wins: $\mathbb{P}[f(V) = +1]$.

→ Maximized by the constant function that always picks +1 🙄

Require **neutrality**: for all v , $f(-v) = -f(v)$.

The Best **Neutral** Voting Rule?

To satisfy neutrality, must decide for each pair $\{\nu, -\nu\}$ of preference profiles which one should be $+1$ and which one -1 .

Neutrality: for all ν ,
 $f(-\nu) = -f(\nu)$.

To maximize probability of returning correct alternative, simply return $+1$ on the profile in the pair that is more likely to occur!

[Derivation on blackboard.]

Best Neutral Voting Rule

We just proved:

Theorem [Nitzan & Paroush 1982]. In this model, the best neutral voting rule is weighted majority rule with weights w_i proportional to $\log \frac{p_i}{1 - p_i}$.

Observation: Voters with $p_i = 1/2$ have zero weight; voters with $p_i < 1/2$ even negative weight!

Best Neutral Voting Rule

10

Theorem [Nitzan & Paroush 1982]. In this model, the best neutral voting rule is weighted majority rule with weights w_i proportional to $\log \frac{p_i}{1-p_i}$.



$$\left(\frac{2}{3}\right) \approx 67\%$$

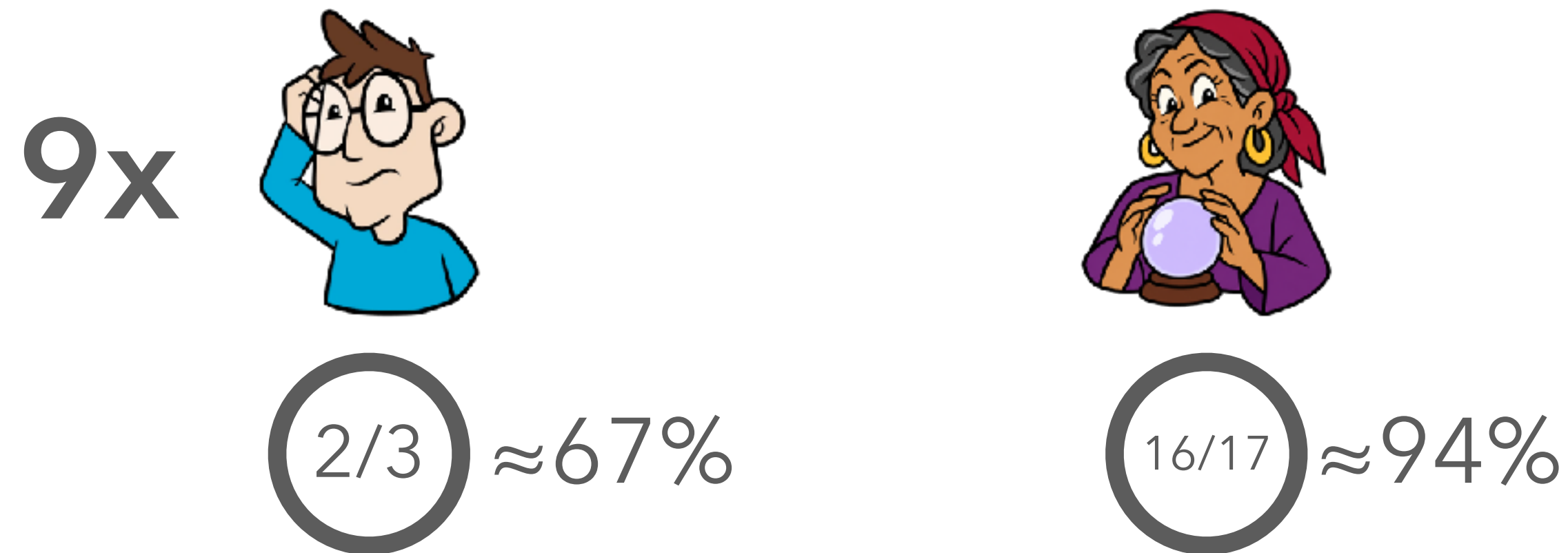


$$\left(\frac{16}{17}\right) \approx 94\%$$

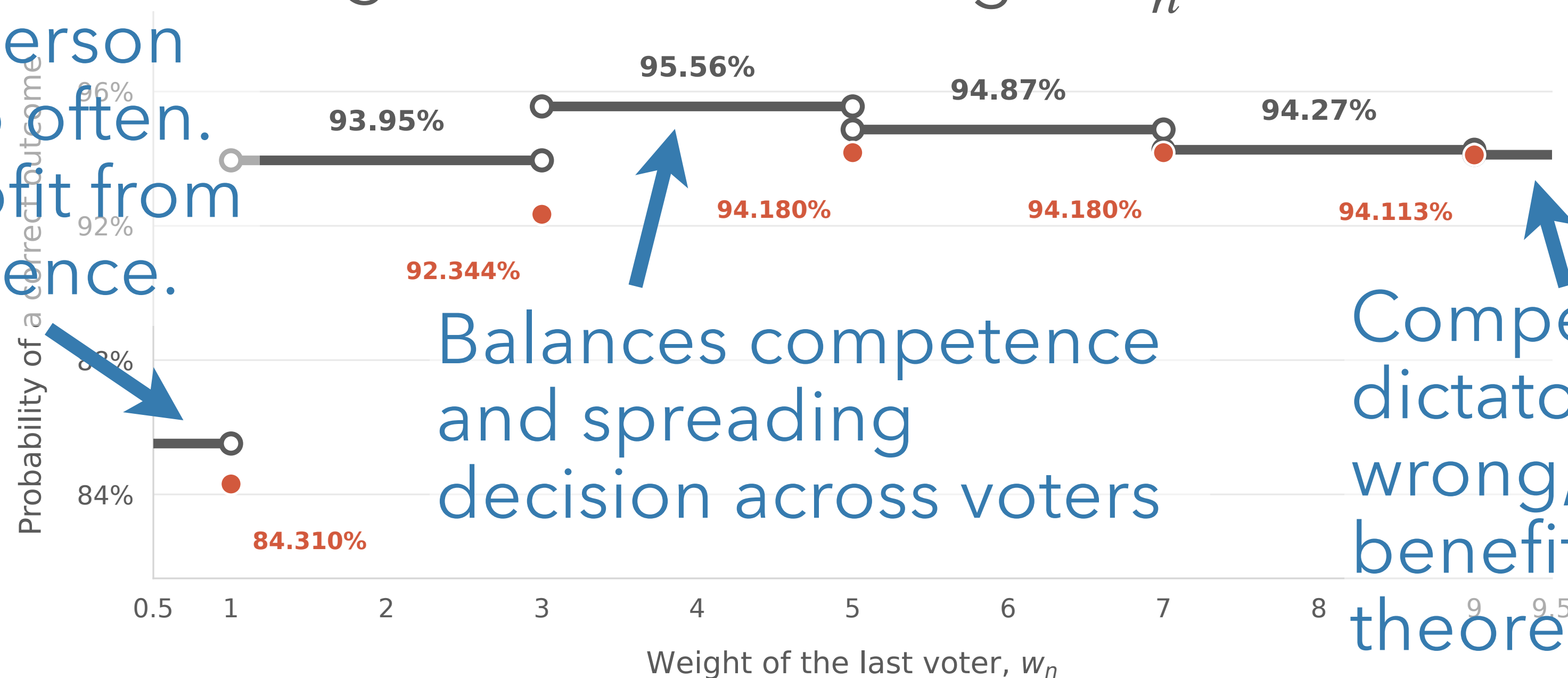
$$4 \cdot \log \frac{2/3}{1/3} = 4 \cdot \log 2 = \log 2^4 = \log 16 = \log \frac{16/17}{1/17}$$

Right-hand side voter should four times the weight of left-hand-side voter.

Tradeoffs



Jury theorem, but competent person outvoted too often. We don't profit from their competence.



Balances competence and spreading decision across voters

Competent person is dictator. If they get it wrong, so do we. Don't benefit from jury theorem.

Reflections on Nitzan & Paroush

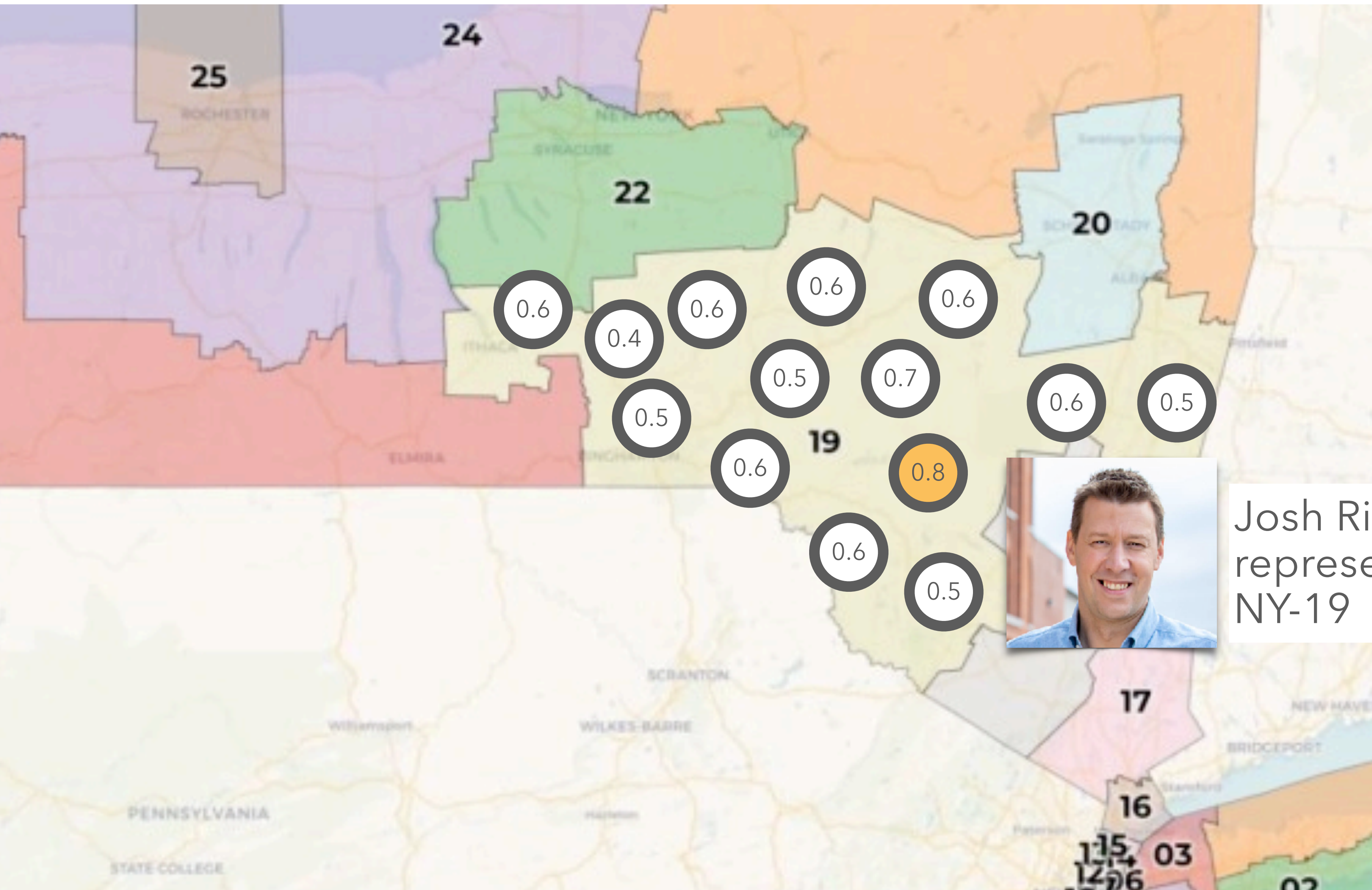
12

This kind of weighted majority voting does not sound very democratic ("one person, one vote").

How would we even know the competences of voters?

Maybe the voters can tell...

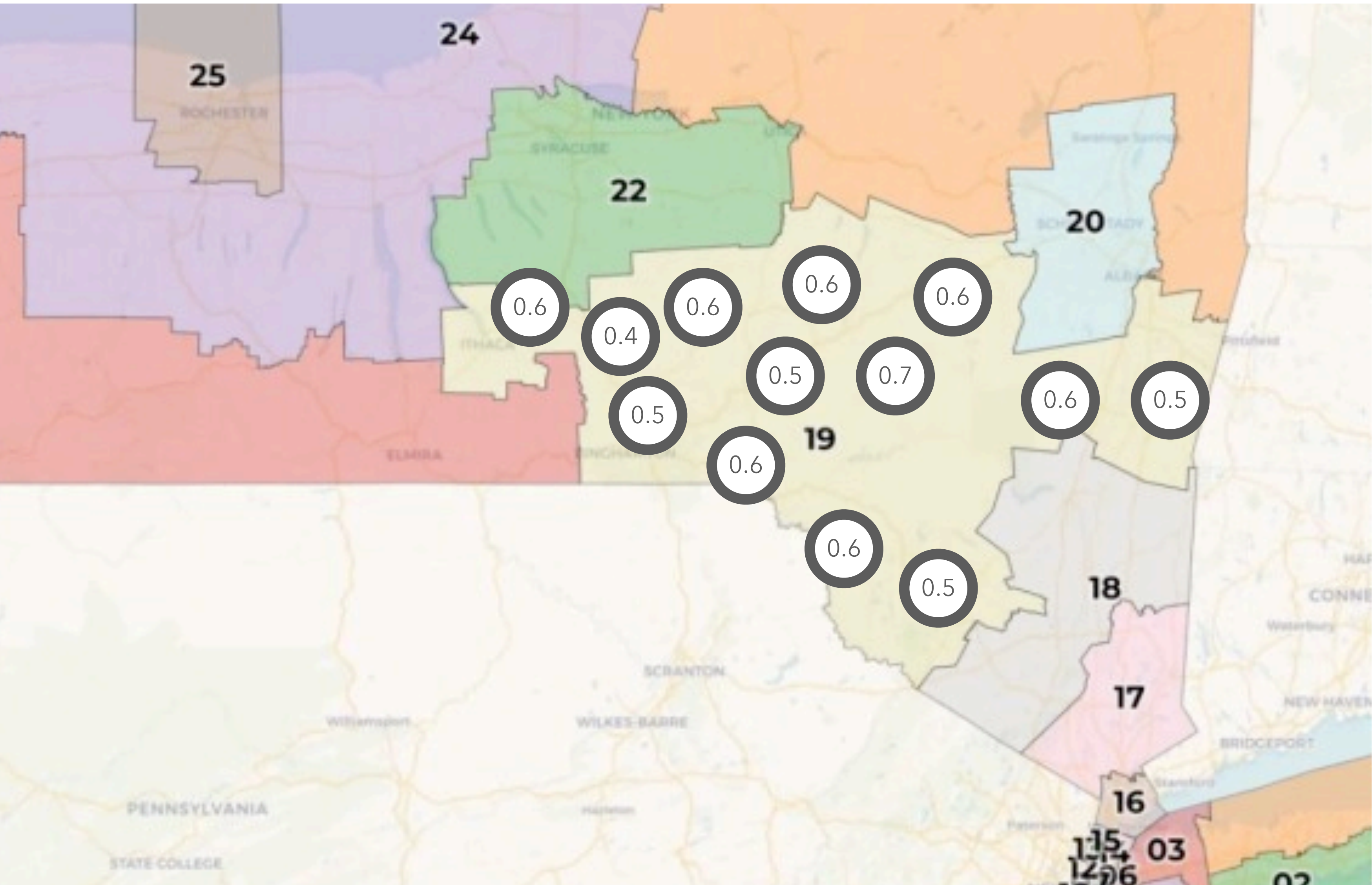
Representative Democracy



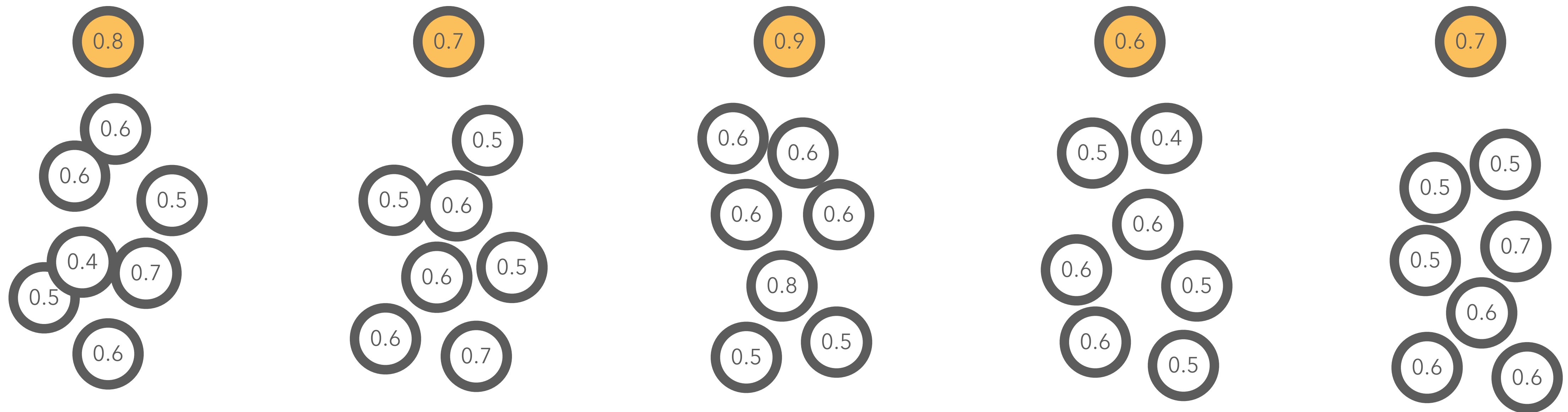
Josh Riley, House representative for NY-19



Representative Democracy



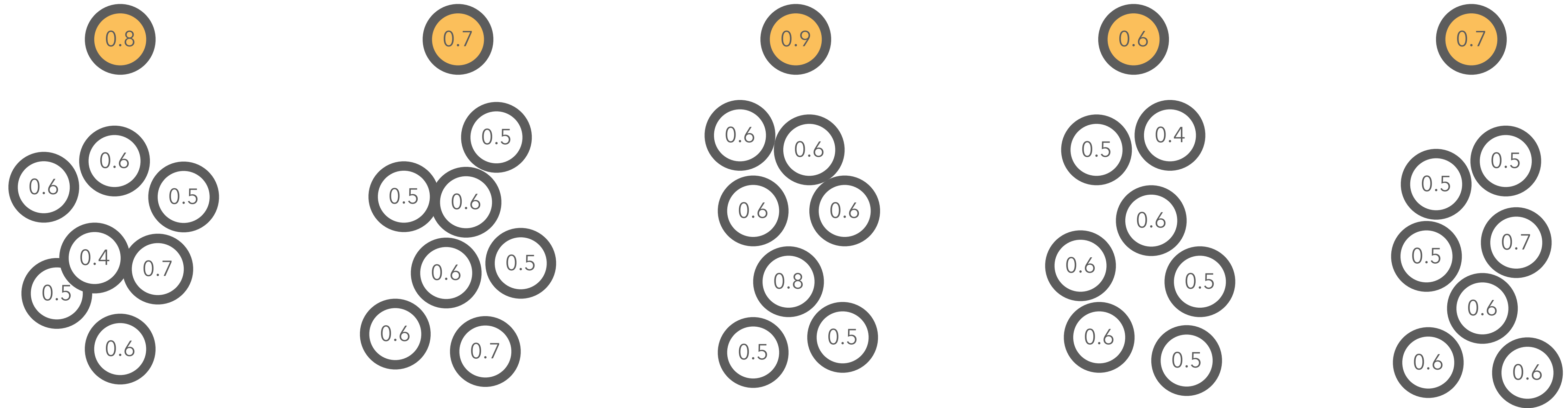
Representative Democracy



When voting on bills, representatives have same weight, everyone else weight 0.

Representative Democracy

16



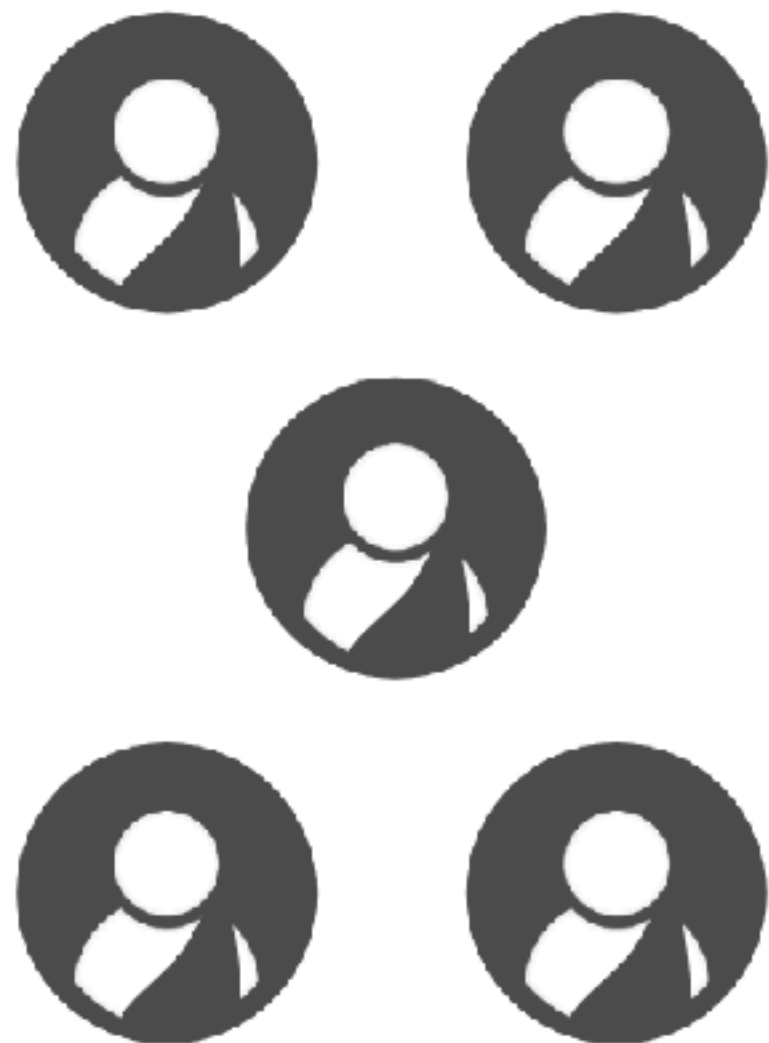
Far from optimal in our model, even if each district picks most competent member.

But weighted voting inherent in delegation:
district members give their vote to representative.

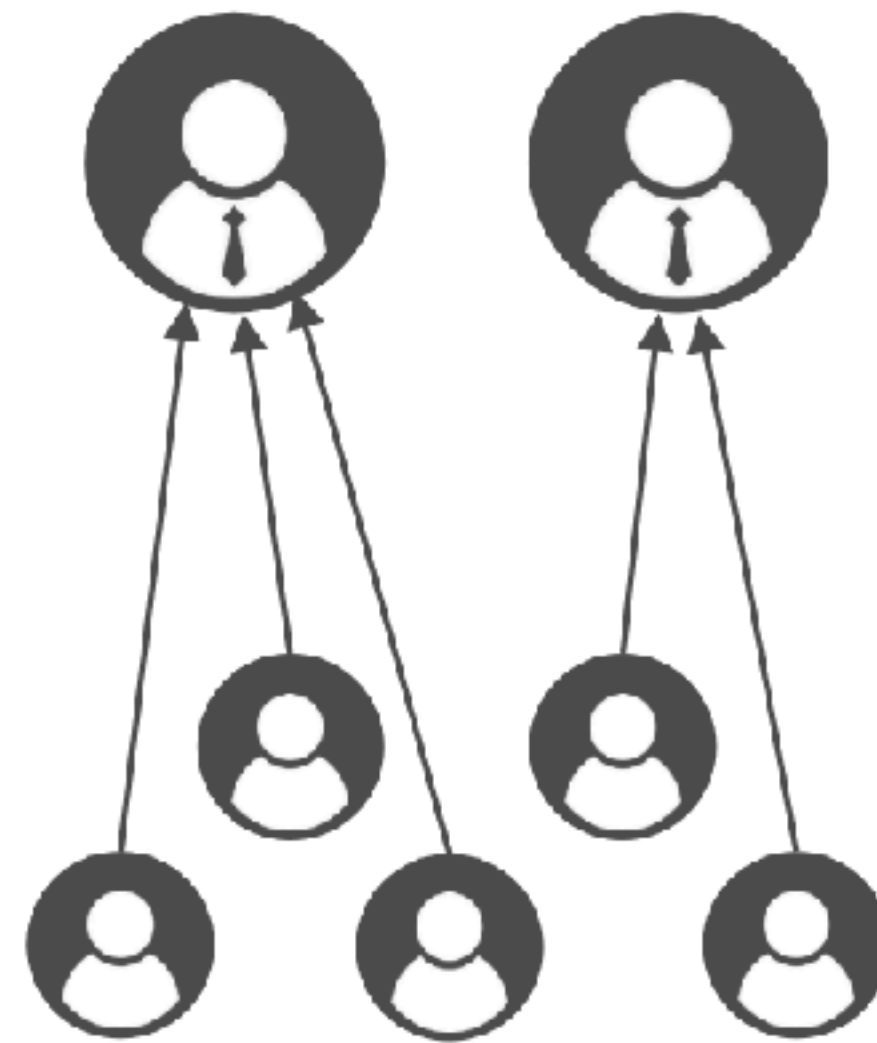
Liquid Democracy

17

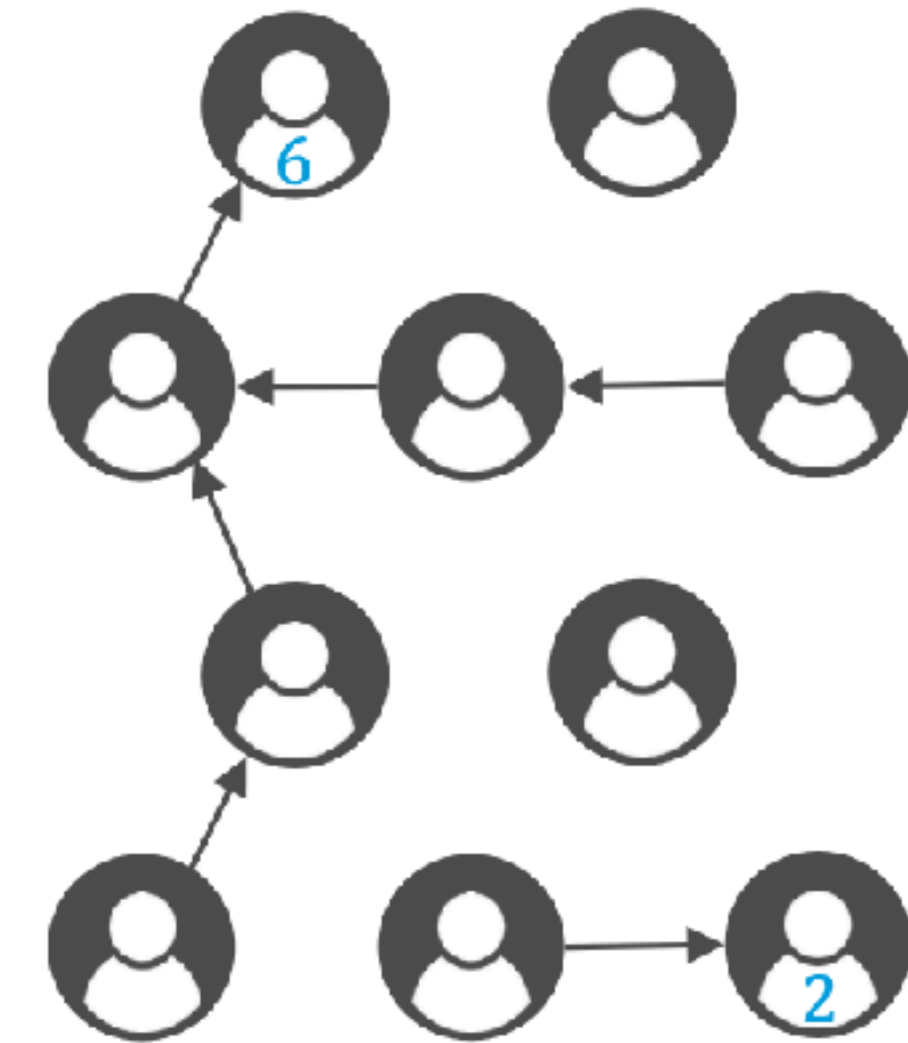
direct
democracy



representative
democracy



liquid
democracy



Idea: flexible individual delegation. Voters can vote themselves, or lend their vote to anyone else.

Guiding Question

Is liquid democracy a way to outperform (unweighted) direct democracy, if voters voluntarily delegate to more competent peers?
Definitely possible, but turns out to be delicate.

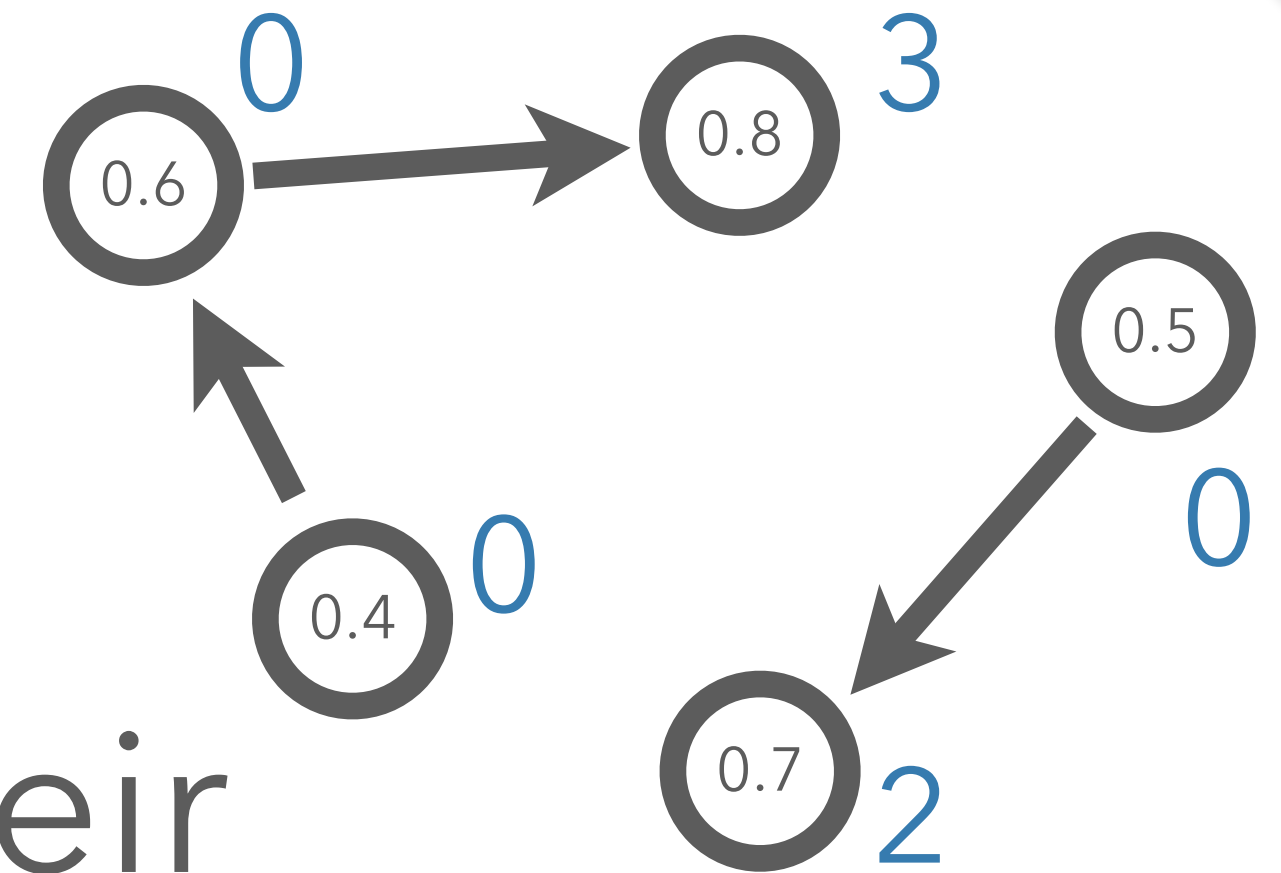
Model for Liquid Democracy

19

As before, voter i has competence p_i , their probability of voting +1.

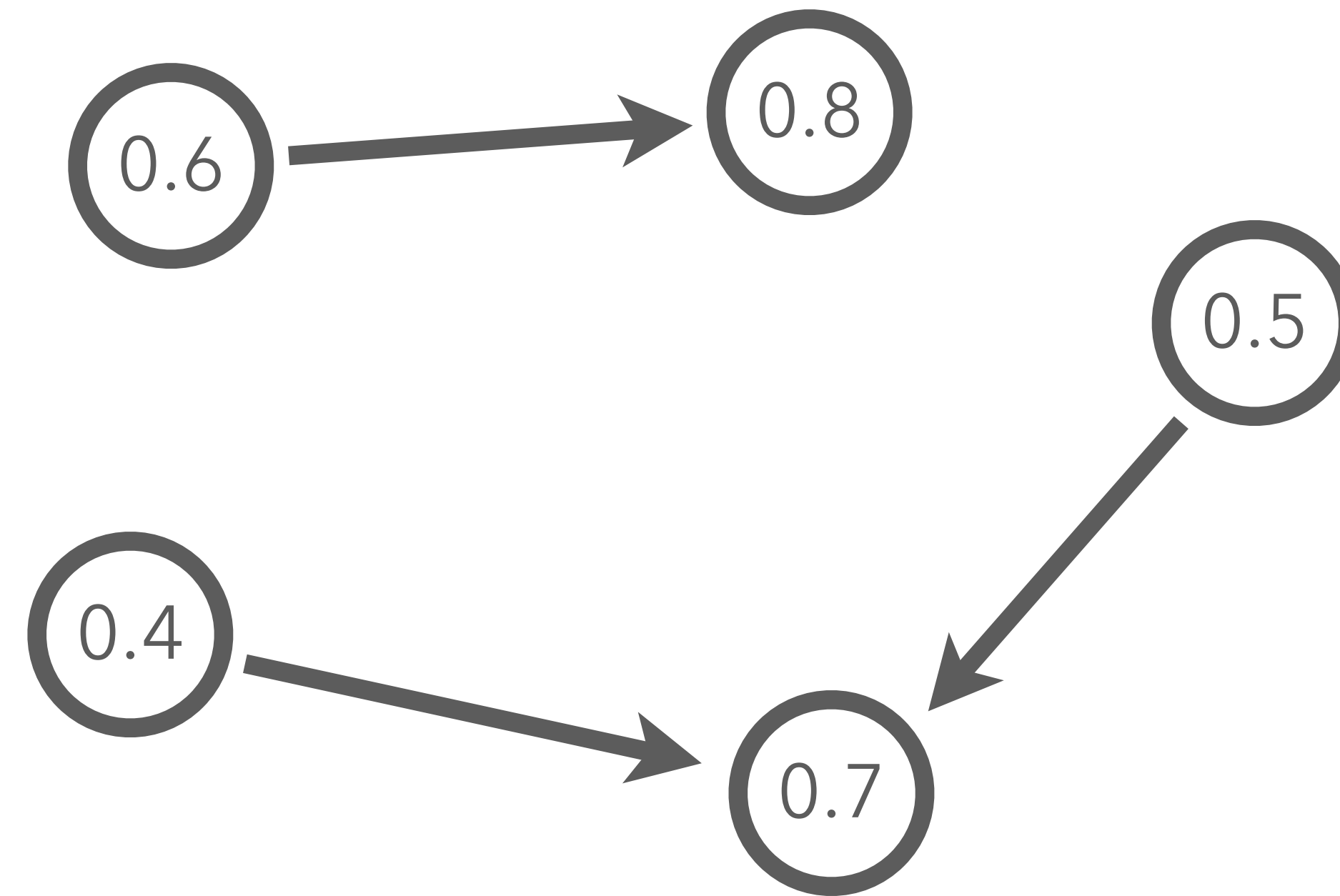
i can vote themselves, or delegate their vote to some voter j (" $i \rightarrow j$ " in directed graph). Votes delegated to i get passed on to j .

Then, weighted majority vote. Delegators get 0 weight, those voting themselves get weight equal to number of delegated votes.



Question 1

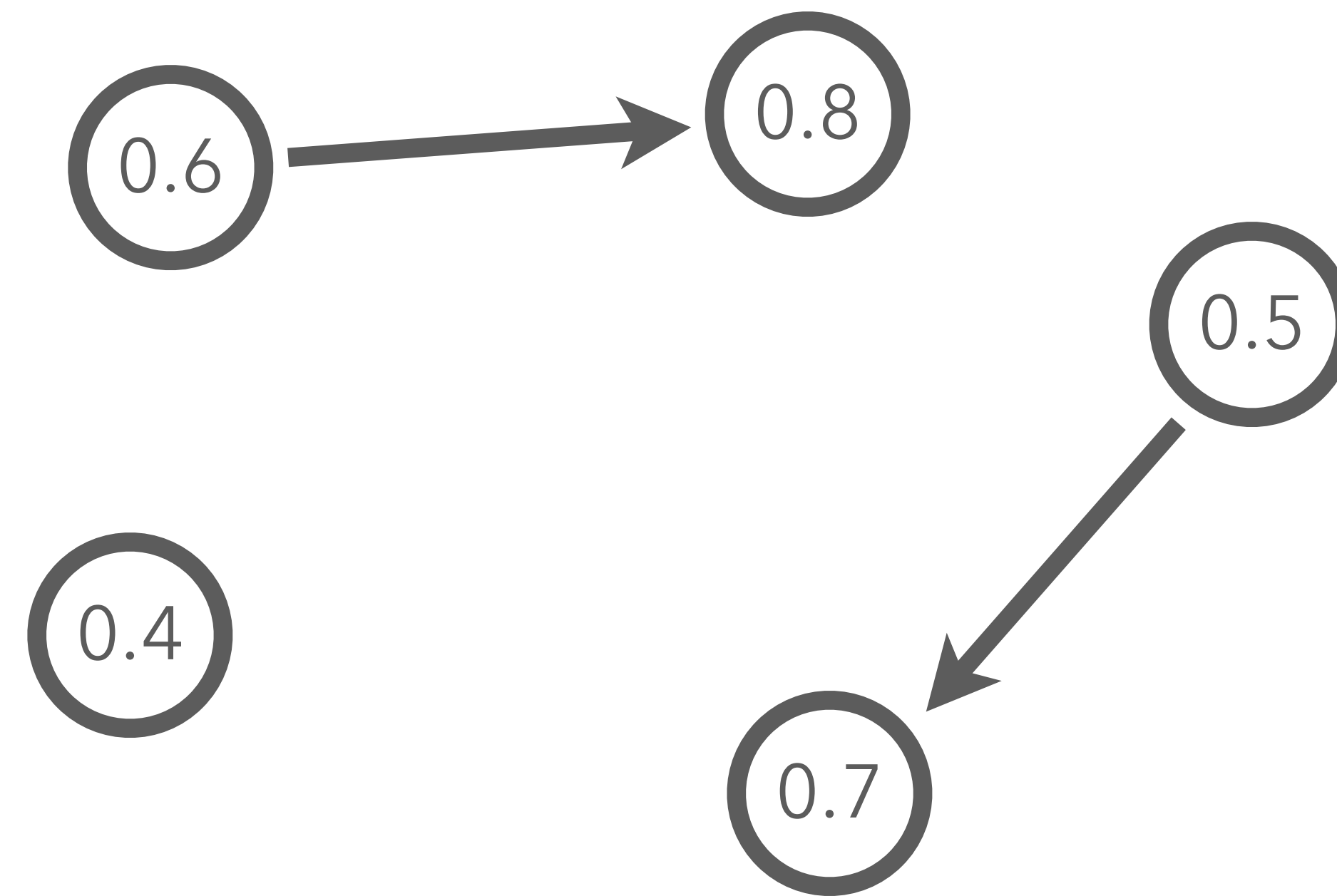
20



Q: What's the probability of the correct outcome +1?

Question 2

21



Q: What's the probability of the correct outcome +1?

Delegation Can Harm



$n - 1$ voters with competence 0.6, one with 0.7.

If they all vote themselves, probability of right outcome goes to 1 as $n \rightarrow \infty$ by jury theorem.

If they all delegate to 0.7, right outcome with probability 0.7.

Delegation Can Harm



Similar tradeoff: Some delegation to higher competence is fruitful. But concentrating decision in too few hands harms decision accuracy.

A Real Issue!

24

www.spiegel.de/international/germany/liquid-democracy-web-platform-makes-professor-most-powerful-pirate-a-818683.html

DER SPIEGEL

Abonnement Anmelden >

Menü < eite > International > Germany > Pirate Party > Liquid Democracy: Web Platform Makes Professor Most Powerful Pirate

Foto: dapd



Fotostrecke

Photo Gallery: Germany's Pirates Promote Digital Democracy

3 Bilder

Liquid Democracy

Web Platform Makes Professor Most Powerful Pirate

Bibliography

Nitzan & Paroush. *Optimal decision rules in uncertain dichotomous choice situations*. International Economic Review 1982.

Kahng, Mackenzie & Procaccia: *Liquid democracy*. Journal of Artificial Intelligence Research 2021.