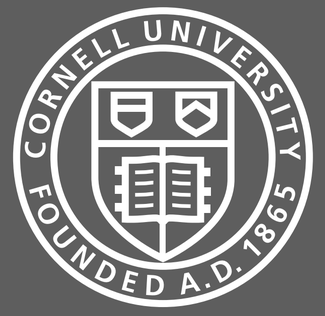
ORIE 4360/5360:

Algorithms for Democracy

Paul Gölz

Lecture 1, ORIE 4360/5360, Fall ’26



Logistics

Syllabus has more details. Problem set 0 posted.

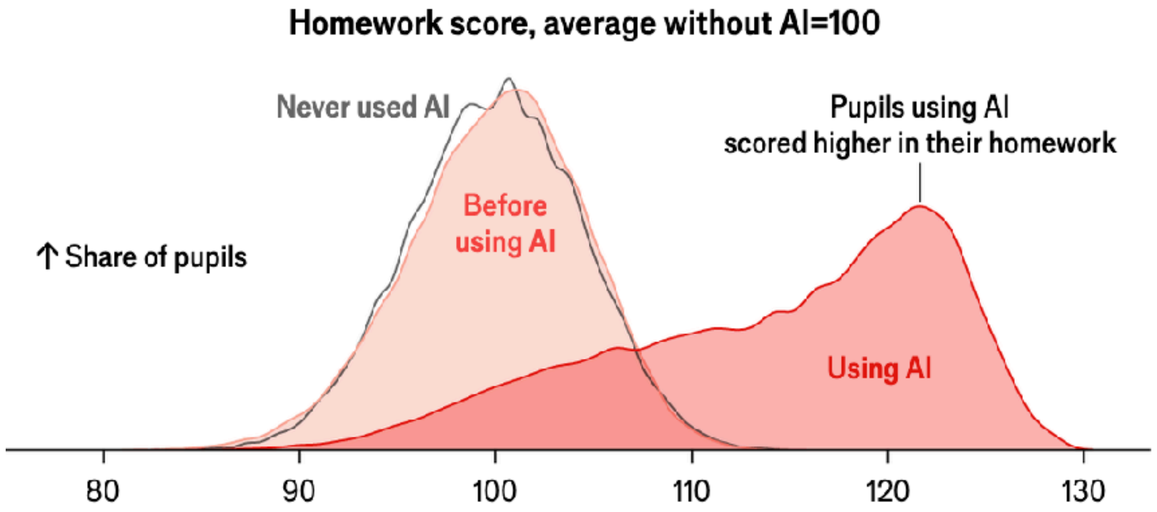
~6 regular problem sets every two weeks.

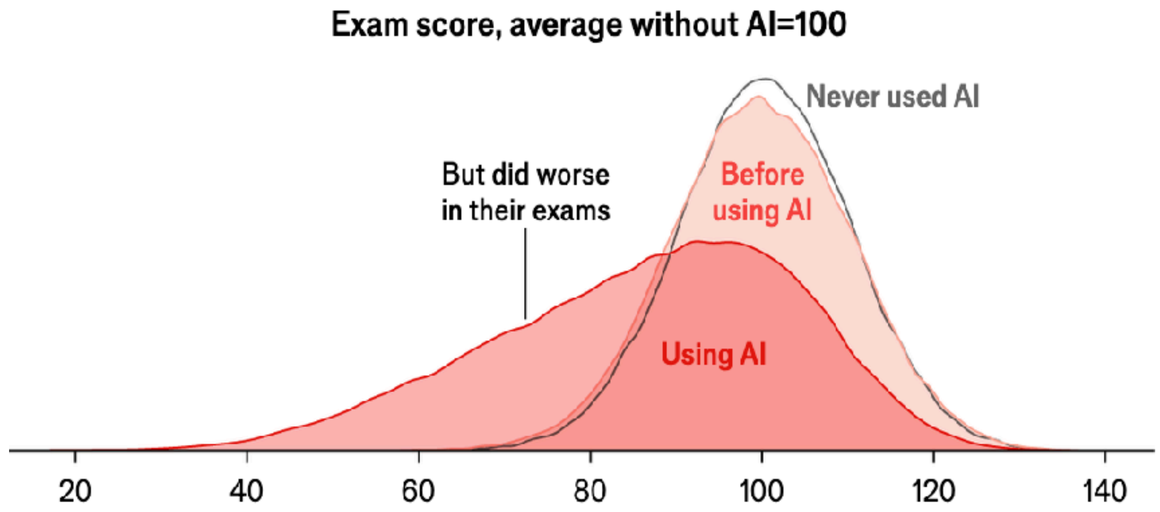
2 in-class prelims (dates TBA) and a final.

No single textbook, many results from recent research. *Handbook of Computational Social Choice* is useful (see syllabus for download). I'm posting slides on https://paulgoelz.de/4360. Last slide has references.

Thoughts About AI

TL;DR: Please don't use AI on homeworks.



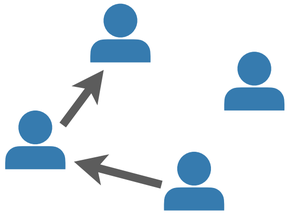
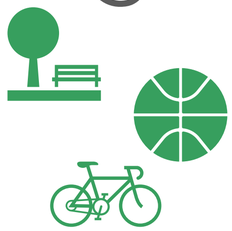


But you can use it as a learning tool: e.g., re-explain a concept; quiz you before an exam, …

Source: Economist and Strömberg et al. (2026): *The Generative AI Learning Penalty*.

What You Will Take Out

- Learn about different group decision settings.

e.g.,  Liquid Democracy  Participatory Budgeting  Citizens' Assemblies

- Learn and practice a range of technical tools.

  e.g., axiomatic method, cardinal welfarism, maximum likelihood estimation, distortion of voting rules.

- See how mathematical and algorithmic tools can be applied to a "soft" topic.
- Sharpen your precision in writing and evaluating mathematical arguments.

What You Will Take Out

…

- Sharpen your precision in writing and evaluating mathematical arguments.

  ↑ My advice for what to train in an age of AI.

  We want to support you in honing your skills, from whatever your current level is.

Check in: Mathematical Arguments

Q: How comfortable do you feel with writing proofs?

pollev.com/goelz

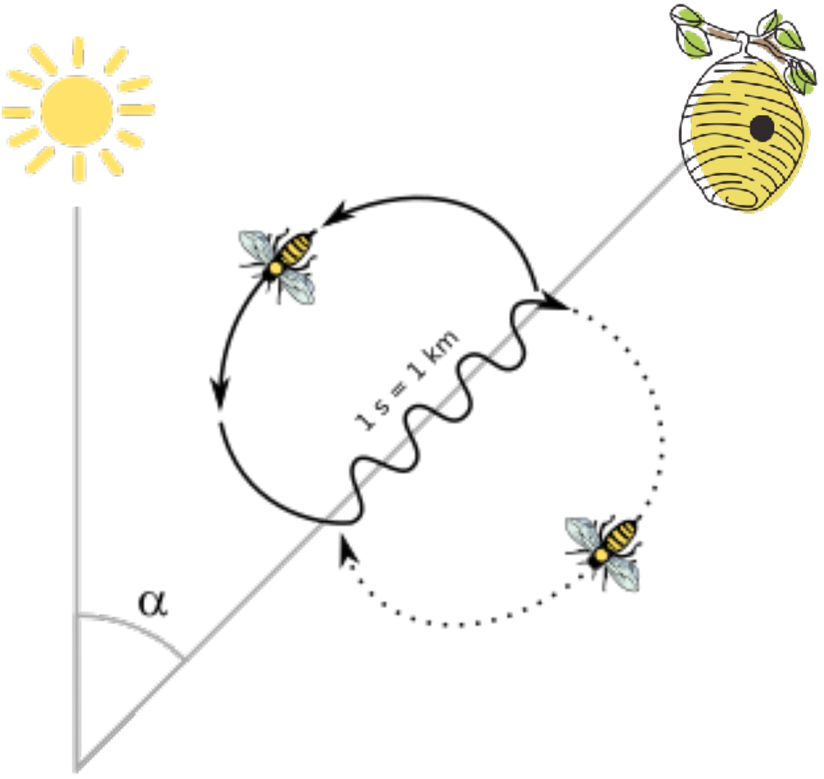
Q: How comfortable do you feel with finding errors in a written mathematical argument?

Please Come to Us!

Gabriel and I want to help you learn & succeed.

Please reach out with any questions and concerns! Come to office hours, chat after class, ask on Ed, ….

Decision Making for Groups



(link, mute sound, start at 4:00)

Voting Model

Voters $N = \{1,2,\ldots,n\}$.

Alternatives $M = \{a, b, c, \ldots\}, |M| = m$.

Each voter i has a ranking $\sigma_i \in \mathscr{L}$ over the alternatives; $a \succ_{\sigma_i} b$ means that i prefers a to b.

A preference profile $\boldsymbol{\sigma} \in \mathscr{L}^n$ is a collection of all voters' rankings.

A social choice function is a function $f : \mathscr{L}^n \to M$; a social welfare function is $f : \mathscr{L}^n \to \mathscr{L}$.

Slide Notation for Preference Profiles

voters numbered $1,\ldots,5$;
each has a column

alternatives
a, b, c, d

1	2	3	4	5	
a	*a*	*b*	*c*	*d*	← *d* is voter 5's favorite alternative
d	*d*	*d*	*d*	*c*	
b	*c*	*c*	*b*	*b*	
c	*b*	*a*	*a*	*a*	← *a* is voter 5's least favorite alternative

The Case of Two Alternatives ($m = 2$)

Ubiquitous: (essentially) US presidential elections, criminal trials, …

Social choice functions and social welfare functions are the same, let's use social choice functions.

1	2	3	4	5	6	7
a	*a*	*b*	*a*	*b*	*b*	*a*

Q: Which social choice function makes sense?

The Case of Two Alternatives ($m = 2$)

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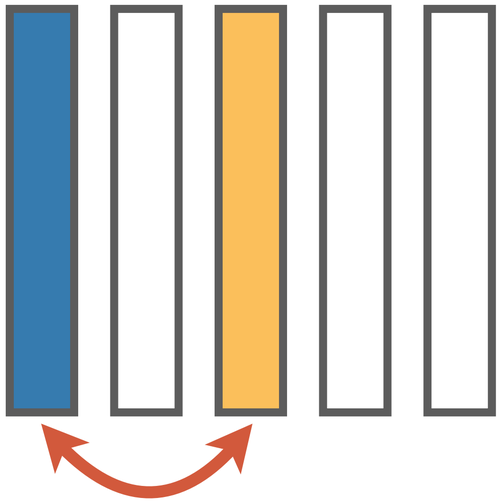
Social choice functions and social welfare functions are the same, let's use social choice functions.

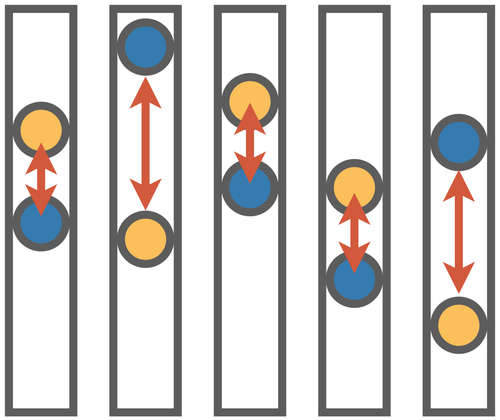
1	2	3	4	5	6	7
a	*a*	*b*	*a*	*b*	*b*	*a*

Q: Can you explain why majority is best?

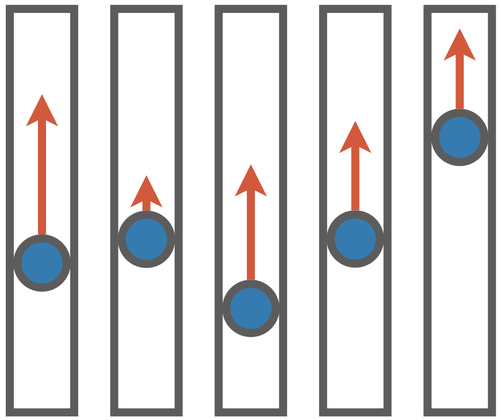
Axioms Satisfied by Majority



Anonymity: Permuting the assignment of voters to rankings doesn't change the outcome.

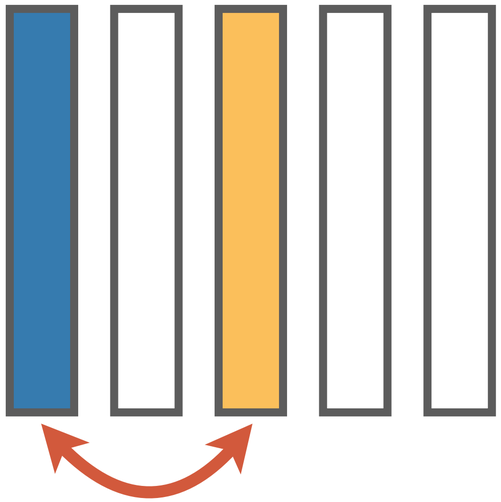


Neutrality: Permuting the alternatives permutes the outcome in the same way.



Monotonicity: Pushing x upward in the votes can't harm x. For $m = 2$: flipping voters from y to x can't flip winner from x to y.

Axioms Satisfied by Majority



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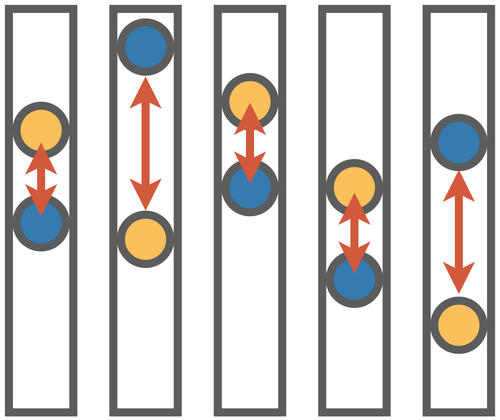

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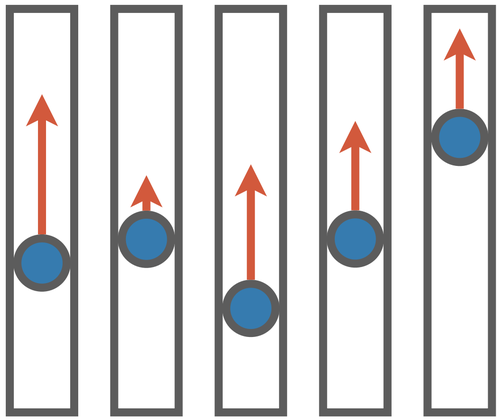


Monotonicity: Pushing x upward in the votes can't harm x. For $m = 2$: flipping voters from y to x can't flip winner from x to y.

May's Theorem

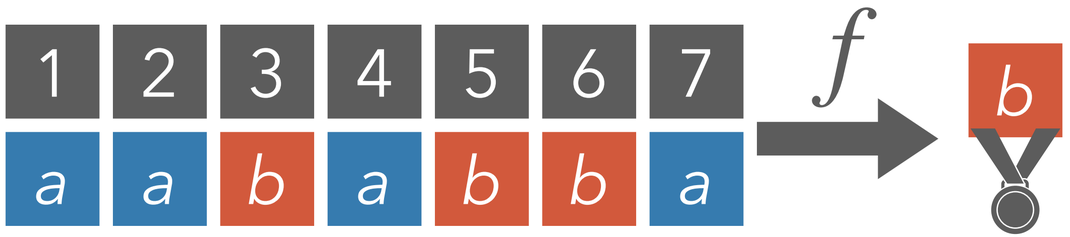
Theorem [May 1952]. Suppose that $m = 2$ and n is odd. Then, $f : \mathscr{L}^n \to A$ is anonymous, neutral, and monotone if and only if it is majority.

(Generalizes to even n, just need to handle ties.)

Q: For $m = 2$, are there other rules that satisfy anonymity and neutrality? anonymity and monotonicity? neutrality and monotonicity?

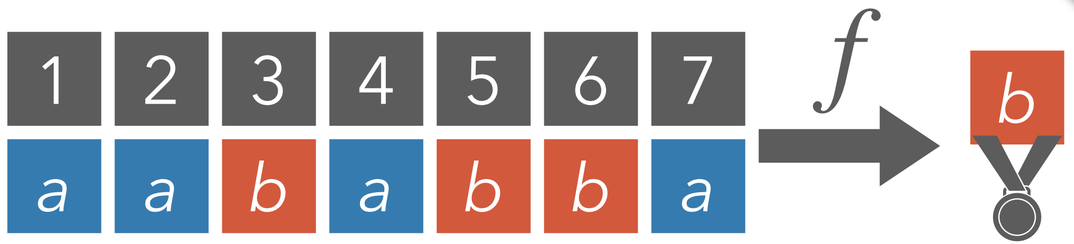
Proof of May's Theorem

We already know that majority satisfies these axioms. So now, suppose for contradiction that our social choice function f is anonymous, neutral, and monotone, but on some profile $\boldsymbol{\sigma}$, some candidate with $t < n/2$ votes is chosen:



Proof of May's Theorem (Cont.)

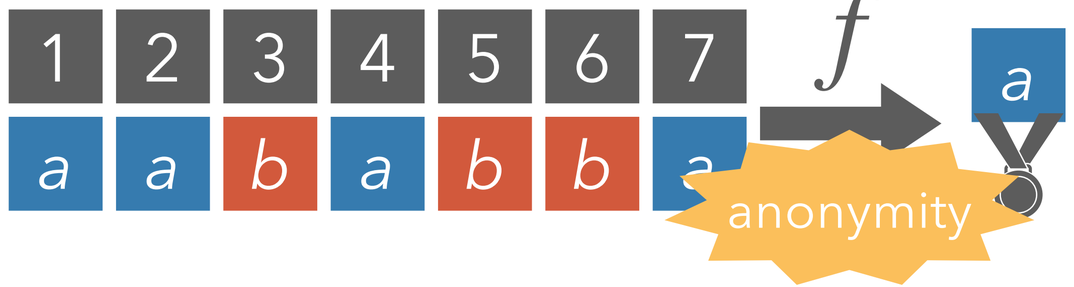
$t < n/2$ voters vote for b $\boldsymbol{\sigma}$:



Flip all votes to obtain $\boldsymbol{\sigma}'$:

Flip $n - 2t$ votes to a $\boldsymbol{\sigma}''$:

Rearrange the voters $\boldsymbol{\sigma}$:



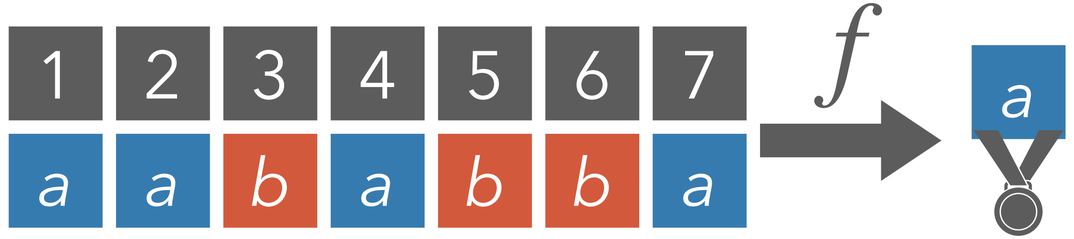
Proof of May's Theorem (Cont.)

$t < n/2$ voters vote for b σ:

Contradiction! ⚡

Hence, f must be majority, QED.

Rearrange the voters σ:



May's Theorem

Theorem [May 1952]. Suppose that $m = 2$ and n is odd. Then, $f : \mathscr{L}^n \to A$ is anonymous, neutral, and monotone if and only if it is majority.

We have proved that, if you want a reasonably sane social choice function for $m = 2$, majority is your only choice. The dream.

Trickier With More Alternatives…

1	2	3	4	5
a	*a*	*b*	*c*	*d*
d	*d*	*d*	*d*	*c*
b	*c*	*c*	*b*	*b*
c	*b*	*a*	*a*	*a*

Q: Who do you think should be the winner?

Not obvious, different rules give different answers. Let's see a few.

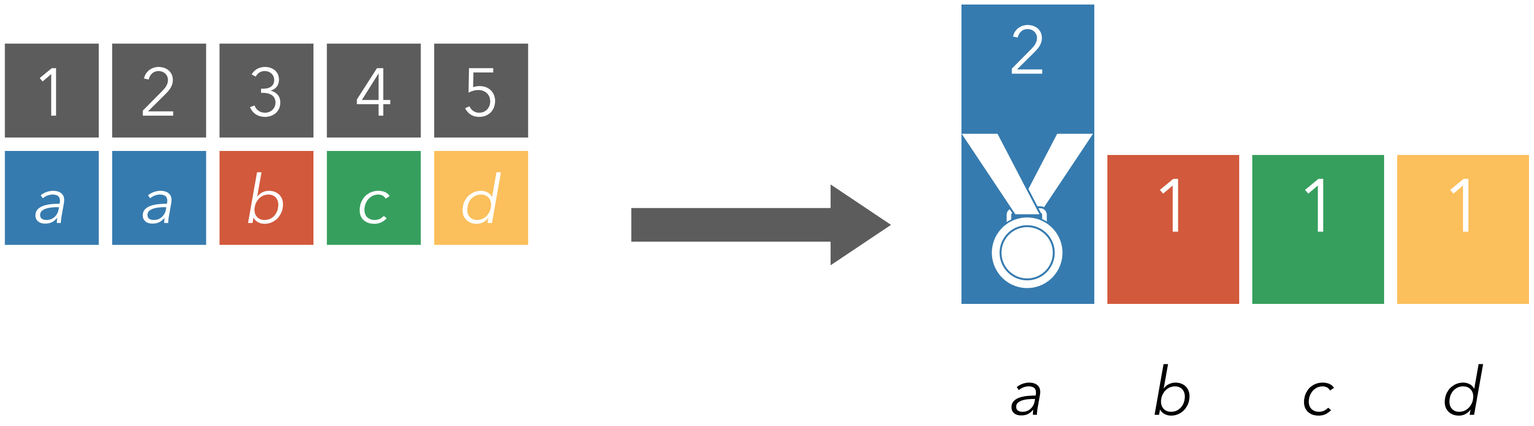
Plurality

Each person votes for their most preferred alternative, and the alternative with the most points wins.

1	2	3	4	5
a	*a*	*b*	*c*	*d*
d	*d*	*d*	*d*	*c*
b	*c*	*c*	*b*	*b*
c	*b*	*a*	*a*	*a*

Plurality

Each person votes for their most preferred alternative, and the alternative with the most points wins.

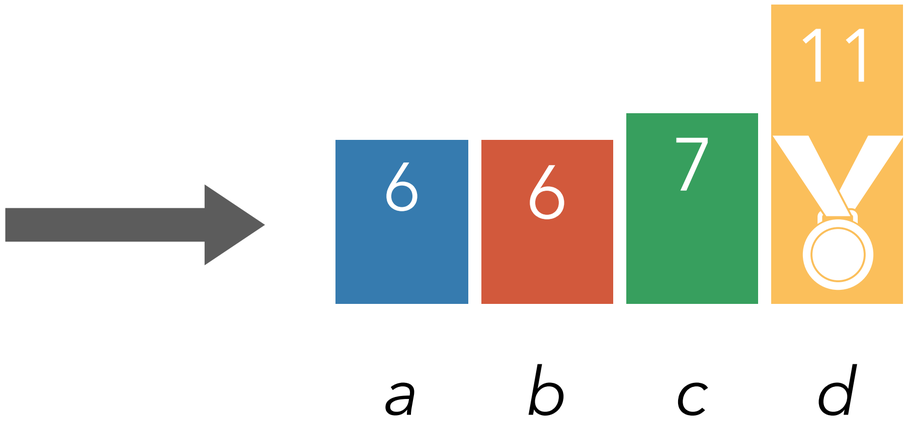


Borda Count

Each voter gives 0 points to their last alternative, 1 point to their 2nd-to-last alternative, 2 points to 3rd-to-last, …. Alternative with most points wins.

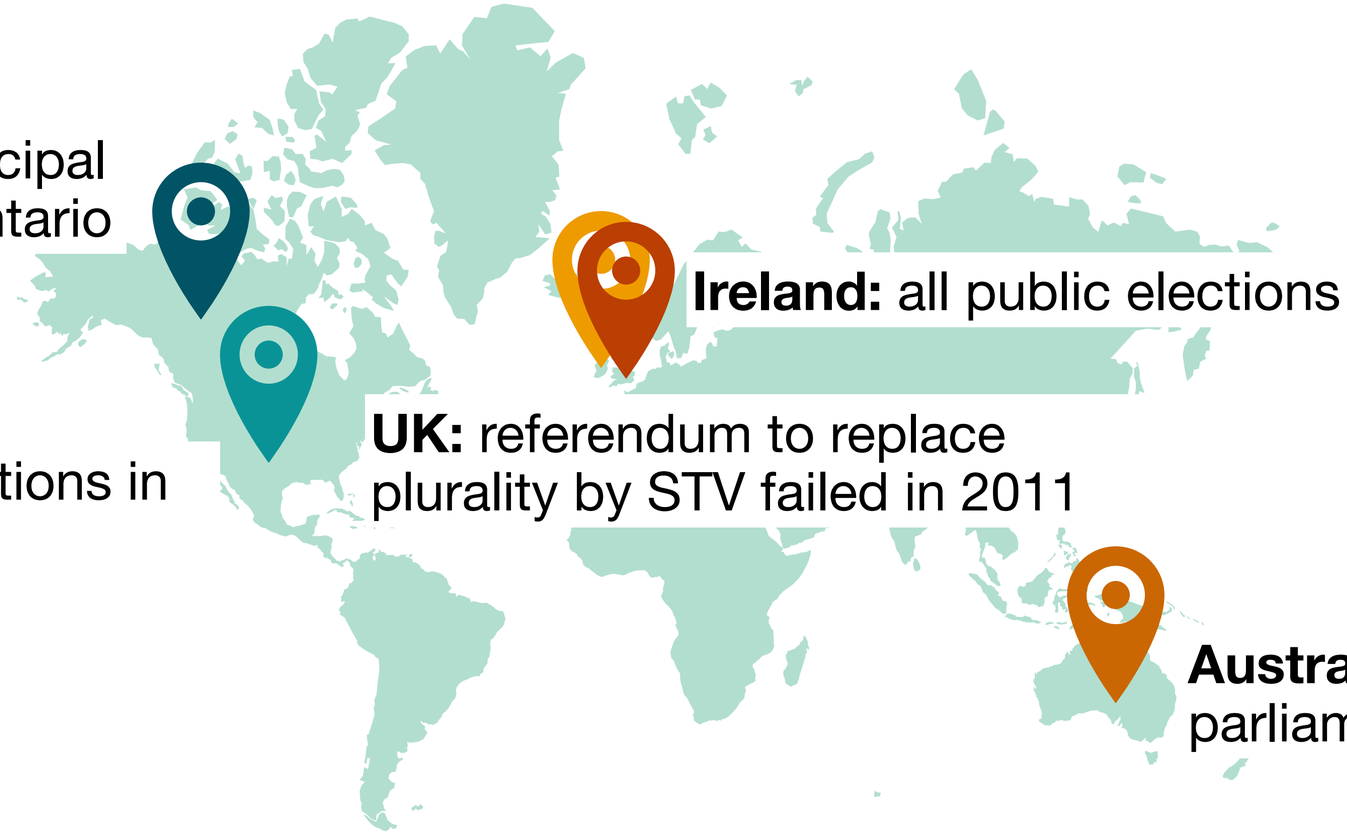
Jean-Charles de Borda, 1733–1799

	1	2	3	4	5
3 points →	*a*	*a*	*b*	*c*	*d*
2 points →	*d*	*d*	*d*	*d*	*c*
1 point →	*b*	*c*	*c*	*b*	*b*
0 points →	*c*	*b*	*a*	*a*	*a*



Single Transferable Vote (STV)

In each round eliminate the alternative with fewest plurality score, last alternative standing is the winner.



Canada: municipal elections in Ontario

Ireland: all public elections

UK: referendum to replace plurality by STV failed in 2011

US: statewide elections in Maine and Alaska

Australia: parliamentary elections

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1	2	3	4	5
a	*a*	*b*	*c*	*d*
d	*d*	*d*	*d*	*c*
b	*c*	*c*	*b*	*b*
c	*b*	*a*	*a*	*a*

1	2	3	4	5
a	*a*	*b*	*c*	
				c
b	*c*	*c*	*b*	*b*
c	*b*	*a*	*a*	*a*

1	2	3	4	5
a	*a*		*c*	
				c
	c	*c*		
c		*a*	*a*	*a*

Plurality

Borda

STV

Which candidate should be the winner?

Which voting rule is the best one?

Bibliography

Chapter 2.1–2.4 in Brandt et al. (2016): *Handbook of computational social choice*.